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T H E O R I G I N O F E A R T H Q U A K E S

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SYMBOLS AND NOTATIONS

$x_i (i=1,2,3)$	Rectangular Cartesian coordinates
ρ	Density of the visco-elastic material (gm/cm^3)
μ	Rigidity coefficient of the visco-elastic material ($gm./cm\ sec^2$)
σ	Newtonian viscosity coefficient of the visco-elastic material (gm./cm.sec.)
$S_{ik} (i,k=1,2,3)$	Components of the stress tensor in the visco-elastic layer ($gm./cm.sec^2$)
$u_i (i=1,2,3)$	Components of the displacement vector in the visco-elastic layer (cm.)
ρ'	Density of the plastic material ($gm./cm^3$)
μ'	Rigidity coefficient of the plastic material ($gm./cm.sec^2$)
σ'	Newtonian viscosity coefficient of the plastic material (gm./cm.sec.)
$S'_{ik} (i,k=1,2,3)$	Components of the stress tensor in the plastic layer ($gm./cm.sec^2$)
$u'_i (i=1,2,3)$	Components of the displacement vector in the plastic layer (cm.)
$\varepsilon'_{ik} (i,k=1,2,3)$	Components of the strain tensor of the Maxwell material
H	Thickness of the visco-elastic layer (km.)

(Symbols and Notations, Continued)

V_p	Velocity of propagation of the wave (km./sec.)
m	Time attenuation coefficient (1/sec.)
p	Frequency of the simple harmonic progressive wave (rad./sec.)
T	Period of the simple harmonic progressive wave (sec.)
$\lambda = 2\pi/f$	Wave length (km.)
c_{31}	Attenuation coefficient in the direction of x_1 in the plastic layer
c_{32}	Attenuation coefficient in the direction of wave propagation.
$\xi = \frac{\sigma'}{\sigma}$	
$\eta = \frac{\mu'}{\mu}$	
$\zeta = \frac{\rho'}{\rho}$	
$\bar{i} = \sqrt{-1}$	

ABSTRACT

In this thesis an attempt is made to demonstrate in accordance with known physical principles that significant changes in the macroscopic equilibrium of the earth can be attributed to rate processes which do not call upon the existence of macroscopic gradients and disturbances in the earth. Such processes and their relation to known mechanisms of plastic deformation and to the experimentally established behavior of materials under high pressures are critically evaluated. This evaluation is carried out in the light of some well-established concepts of statistical mechanics and modern physics. In so doing specific methods for producing seismic disturbances which are based upon known mechanisms of plastic deformation and rupture become indicated. It is also shown that under very high pressures, pressure and temperature can complement each other in producing disturbances associated with polymorphic transitions of materials leading to a reduction in symmetry of their lattice structure.

A specific mechanism for producing and repeating earthquakes at shallow and intermediate depths is proposed. This mechanism is based upon the existence of a visco-elastic surface layer supported by a plastic material embodied with stress relaxing properties.

The effect of such a configuration upon the propagation of Love Waves is investigated quantitatively. This investigation shows that the plastic sub-layer would explain the observed selection of the period of Love Waves.

I. Introduction:

The fundamental work of H. Benioff (1) clearly establishes that the elastic-rebound strain increments associated with individual earthquakes in a given fault are typical of the creep recovery characteristics of rocks as measured in the laboratory. These observations suggest that tectonic processes and earthquakes may be more generally and completely determined by relaxation phenomena of various types taking place within the materials of the earth. In this thesis an attempt is made to evaluate this idea and to investigate quantitatively some of its implications. In so doing a specific mechanism for the origin and repetition of earthquakes and related tectonic activity is suggested and proposed.

The physical constitution and state of the earth at a given time and its change with respect to time, are determined by the physical properties of the materials of the earth and their physical environment as functions of time. Consequently changes of state of the earth are, in general, determined (a) by atomic and molecular changes altering the physical properties of its materials (which will be referred to henceforth as changes on the microscopic scale) and (b) by changes of the mechanical and thermodynamic state of its material on the macroscopic scale, caused by macroscopic disturbances of their physical environment.

To evaluate the relative importance of these causes for changes in state of the earth manifested on a macroscopic scale, is an open problem. This thesis attempts to evaluate, in part, macroscopic effects that can be attributed to changes on the physically microscopic scale. To accomplish this one must consider (a) the possible mechanisms for producing such changes, (b) the source of energy that makes these possible, and (c) establish a linkage between the time changes occurring on the microscopic and macroscopic scales so that the macroscopic effect can be evaluated analytically. This linkage is suggested by a study of various methods for representing qualitatively the microscopic and macroscopic properties of mechanical systems.

Since the changes appear to us on the macroscopic scale, it is natural to first call upon strictly macroscopic phenomena to explain them. Accordingly, this has been the basis for both qualitative and quantitative physical investigations that have been carried out thus far. In these investigations difficulties are encountered either in explaining the origin of large macroscopic disturbances in the earth or in explaining how these changes are repeated in the course of time, consistently with the second law of thermodynamics. These investigations treat the materials of the earth as a continuum; thereby considerably restricting the physical phenomena which they can represent analytically, especially when they are not supplemented with the consideration of related molecular and atomic phenomena.

The investigations (2) pertaining to the calculation of stresses in materials capable of sustaining shearing stresses, consistent with the hypothesis of isostasy of H. H. Pratt (3) and J. F. Hayford (4), treat the materials in the compensating layer as perfectly elastic thereby admitting an infinite number of solutions to the equations of equilibrium. These equations can be rendered mathematically determinate only by introducing arbitrary assumptions in the analysis. If the materials are permitted to undergo plastic as well as elastic deformations, it then becomes possible to make the analysis mathematically determined in some cases, by introducing suitable yield conditions for the plastic deformation. The inherent lack of determinateness in an analysis that treats the materials in the compensating layer as perfectly elastic and the additional determinateness rendered to the analysis by assuming plastic deformation as well, strongly suggest that these materials in general have both elastic and plastic properties.

From an atomic and molecular point of view, the elastic properties of a material are determined by elements of molecular and atomic structure which upon being alternately loaded and unloaded, naturally return to the original unloaded configuration. The elastic properties are generally attributed to deformation of the molecular bonds and not to reorientation of the molecules within the material. If the reorientation time of the atoms and molecules is very large compared to the time for which the deformation mechanism of a material is being considered, then during this time, the material can be considered elastic. The re-

orientation time is in general a function of temperature and the magnitude and direction of the forces applied to a material, and consequently assigning an elastic property to a material, is an idealization, valid only under special conditions. Bearing in mind this idealization, it can be asserted that the mechanism for elastic deformation is time independent and can therefore be described macroscopically by a time independent function relating stress to strain in a one to one correspondence. The analytical investigations referred to above and others have in general treated the materials of the earth as an elastic continuum, where deformation properties are characterized by a time independent stress-strain relationship, and pertain to the calculation of stress and motion (if any) consistent with the equilibrium of forces associated with the macroscopic deformations and other physically macroscopic phenomena. Their investigations, however, do not account for macroscopic changes in equilibrium caused by material changes on a microscopic scale.

Plastic deformation is determined by and consists of microscopic structural changes in materials which take place in the course of time when constant forces are applied to such materials. These structural changes involve reorientation of atoms and molecules in the material and usually permit the material to continue to deform macroscopically for a significant time after the forces are applied. There are in general two principal types of plastic deformation. The first, which is more typical of crystalline materials (metallic and non-metallic), is a deformation that is time dependent but such that the strain approaches a limiting value for a given stress. The second is

predominant in amorphous materials that behave as a viscous fluid where the stress is proportional to the rate of strain and is independent of the strain. Consequently, in amorphous materials the strain does not approach a limiting value for a given stress. Instead, for a given stress the rate of strain is essentially constant. From these observations it is clear that the physics of plastic deformation and its mechanisms depend upon time changes taking place on a microscopic scale. Thus, in attempting to determine the effects of plasticity upon the macroscopic equilibrium of the materials of the earth, it is necessary to refer to a microscopic as well as a macroscopic scale in the analysis and attempt to establish some relationship between them. When the materials are treated as perfectly elastic the microscopic scale is eliminated by introducing a relation between stress and strain based upon Hooke's Law, where both are physically macroscopic quantities and are considered independent of any changes that take place in the microscopic scale within the material. This procedure can usually be justified if the stresses and temperature of the material are sufficiently low so that the material does not yield. However, the effect of the yield and subsequent deformation upon the macroscopic equilibrium cannot be satisfactorily interpreted without taking into account the time changes in the microscopic scale.

In order to investigate analytically their effect upon macroscopic equilibrium it is necessary to represent mathematically the essential features of plastic deformation, expressed in terms

of physically macroscopic quantities. This can be done for a particular substance by determining empirically a relation between stress, strain and time, and using it in the analysis. To determine such a relationship empirically usually leads to many technical difficulties and consequently the tendency has been to more or less arbitrarily assume such relationships, apply them analytically, and check their validity by indirect experiments. This procedure has introduced numerous so-called ideal plastic materials into the literature. These relations, which might be considered as analogous to Hooke's Law for perfectly elastic materials, contain terms which in general represent elastic, quasi-plastic, and perfectly plastic elements in series or in parallel and consequently, together with the equations of equilibrium, do not in general lead to a mathematically determinate system. Furthermore, their coefficients are in general functions of microscopic fluctuations in the material and constitute an essential linkage that permits physical analysis of the effect of microscopic changes upon macroscopic equilibrium. Since from a mathematical point of view introducing time dependent stress-strain relationships leads in many cases to a mathematically undetermined system, it is helpful to consider other ways of determining the stresses and whether or not conditions of quasi-static equilibrium can be satisfied in materials which in part deform plastically. By introducing yield conditions the analysis can in some cases be made mathematically and consequently physically determinate. Such yield conditions have been proposed by H. Tresca (5), St. Venant (6), Coulomb (7), R.v.Mises

(8), H. Henky (9), and others. The only sound criterion for selecting one or another yield condition to be used in an analysis is essentially empirical. However, it is significant to attempt, if possible, physical interpretation of a yield condition used which in general requires considerations on the microscopic scale.

This thesis is divided into seven parts including the introduction. Part II considers the general problem pertaining to the description of the physical "states" of the materials of the earth. It proposes criteria for distinguishing between the different states of these materials. In Part III mechanisms and properties of plastic deformation in single crystals, amorphous material and polycrystalline materials are discussed from the point of view of their geophysical implications. Part IV proposes a specific mechanism for causing and repeating seismic disturbances in the interior of the earth. Some general causes for seismic disturbances, based upon the considerations set forth in Part III, are suggested without presenting specific mechanisms associated with them. In Part V a quantitative analysis of some observable implications of the mechanism proposed in Part IV is presented. In this analysis the effect of a plastic sublayer upon the propogation of Love Waves is investigated. Part VI is an evaluation of the geophysical significance of the seismic mechanism and some of its implications presented in Parts IV-A and V, respectively. Part VII presents a summary and conclusions pertaining to the entire thesis.

II. On the Physical States of the Materials of the Earth:

A fundamental problem in geophysics is to infer from physical principles and measurements supplemented by geological and astronomical data, the physical states of the materials in the interior of the earth. Physical measurements are both necessary and crucial for any attack upon this problem, provided that they can be explained and interpreted by means of an adequate theory. The mathematical formulation of such a theory requires however, a knowledge of the physical parameters which significantly determine the macroscopic states of the materials of the earth at various depths and some understanding of the relation between these parameters and the microscopic phenomena that exist within these materials.

It is clear therefore that, in principle, this technique cannot provide a direct solution to the problem stated. It does, however, suggest a heuristic method for treating the problem. That is, to make judicious postulates about the physical status of the materials of the earth and then study their measurable implications mathematically in accordance with physical principles. Discrepancies between calculation and measurements would require the modification and possibly the use of entirely new postulates. If the significant macroscopic parameters are known and are incorporated in a mathematical analysis, consistent with physical principles, then the macroscopic phenomena within and on the surface of the earth should in principle be predictable at least qualitatively by such a theory. Once a qualitative agreement between theoretical prediction and measurements is established, data can be used for determining in part or completely the

numerical values of the material parameters which appear in the analysis. The extent to which the data can thus determine the material constants will depend upon their nature and scope. The data and sources of information that can be called upon for this purpose are: (a) data obtained from seismic measurements, (b) high temperature and high pressure data obtained in the laboratory, (c) gravitational data, (d) data provided by field geology, structural geology, petrography and mineralogy. Seismic data in particular contain considerable information which is not discernible without adequate mathematical-physical analysis, to bridge the physical state of the materials in the interior of the earth to the physical observables on the surface. As has already been indicated, a necessary step in this direction is to determine the physical parameters which represent the essential macroscopic physical properties of the materials of the earth at various depths. This is primarily a physical problem bearing upon the relation of the atomic, molecular and crystalline structure of a material to its macroscopic physical properties. Although this problem is far from being solved there are nevertheless some reliable empirical results and related theories available which suggest mechanisms for storing the potential energy released periodically in earthquakes that originate at intermediate and great depths. The result of these physical investigations which are considered in some detail in Part III of the thesis, indicate that treating the materials of the earth as perfectly elastic neglects some of the most fundamental physical phenomena connected with the storing, release and propagation of seismic energy. The

fact that shear waves are known to propagate at depths as great as 2900 Km does not necessarily imply that these admissible waves and their character are principally determined by the elastic properties of these materials or that the materials are in an essentially solid state. The propagation of shear waves in a material does imply, however, that it is not a pure fluid and that elastic elements do exist on a macroscopic scale. The mathematical analysis in Part V of the thesis shows that the mechanism which governs the propagation of surface shear waves is significantly affected by the plasticity of the sub-layer in a two-layered configuration. The quantitative results so obtained explain the strong selectivity in the period of Love Waves and indicates how seismic data can be used to calculate some of the material constants of the upper and lower layers.

Before considering questions pertaining to the state of the materials of the earth it is necessary to establish criteria for distinguishing and classifying different states of matter according to available knowledge of its microscopic structure and the physics of the solid state. Many of the ambiguities in geophysics, arising from the use of different terminology and definitions for describing different states of matter, can be removed if criteria for distinguishing states of matter are determined by the actual physical aspects of its structure rather than from a strictly phenomenological point of view. Although there still exist some open fundamental questions relating to the atomic structure of matter and to the physics of crystals, what is now known about these phenomena nevertheless suggests significant

criteria for distinguishing between different states of matter.

In the light of these criteria it becomes clear that for the materials of the earth to be only in one state or another is a rare exception rather than the rule. That is, from the physical insight rendered by these criteria it appears very probable that most of the materials of the earth are agglomerates of both solid and liquid components (in the large sense). The existence of such materials in the earth implies that their macroscopic properties can be changed in the course of time by a heat bath, in the absence of temperature and pressure gradients. As will be shown, the geophysical implications of this conclusion are far reaching and fundamental.

In the light of present knowledge of physics the physical states of a molecule crystal and solid are virtually the same. Each of these is characterized by the existence of solidifying (London-Heitler (10) forces between their atoms. From this point of view it appears meaningless to speak of a strictly amorphous solid since strictly amorphous materials are essentially liquids or gases which are distinguished by the absence of solidifying forces between their molecules, i,e., between atoms of distinct molecules, and a lattice structure. According to Schrödinger (11) the following scheme establishes a physically significant distinction between two states of matter consistent with the actual aspects of the structure of matter:

molecule = solid = crystal

gas = liquid = amorphous

The reality of the solidifying forces that exist in a real solid with a crystalline structure is established by the existence of a critical melting temperature and latent heat of melting at which an abrupt transition (associated with the rupture of homopolar bonds) to the liquid state occurs. Such a critical temperature does not necessarily accompany a change from a liquid to a gaseous state.

An amorphous substance (absence of crystalline structure) which appears solid on the macroscopic scale discloses the absence of a well-defined melting temperature and latent heat of melting. Given time, it flows as a liquid under its own weight at room temperatures. It is therefore much more meaningful, from a physical point of view, to regard such a material as a highly viscous liquid rather than as a solid.

The distinction that has real physical significance in the structure of matter is whether or not the atoms or ions of a material are connected by (homopolar bonds) solidifying forces. In a solid they are, in a liquid and gas they are not.

The elasticity and rigidity of a material are physical properties which are essentially determined by the existence of solidifying forces between its atoms which are arranged in a lattice structure. This was demonstrated by A. Joffe (12) who found experimentally that a well-defined destruction limit exists when a single crystal is subjected to a sufficiently large shearing

stress. In these experiments the lattice structure of the crystal was observed by its X-ray diffraction pattern which changes abruptly when the critical stress is reached. This critical stress was found to depend to some extent upon the rate of loading. This effect becomes less important as the temperature is increased. Accordingly Joffe found that at a temperature of 500°C the X-ray pattern of a rock salt crystal did not change in twenty-four hours when the applied load was 2% less than the measured critical load and changed abruptly when it was 2% above the critical load. The stress at which the pattern changes suddenly is called by Joffe "the destruction limit of the lattice". The existence of such a well-defined limit is further experimental confirmation of the fact that the atoms of a crystal lattice are bound by special (solidifying) forces, i.e., London-Heitler forces.

It was found that the destruction limit of a single crystal is accompanied by a breakdown of the crystal into several parts, although macroscopically the resulting agglomeration appears as a single unit. This breakdown of a single crystal into several parts determines one of two mechanisms for plastic deformation which are discussed in some detail in Part III. Once the destruction limit of a single crystal is passed and it becomes an agglomeration consisting of smaller crystals (each having the same lattice structure as the mother crystal) any further deformation will be plastic and therefore irreversible upon relieving the load. This implies that the forces which act between the atoms of distinct crystals are fundamentally different from the solidifying forces that exist between the atoms within each crystal.

Hence, materials consisting of many crystals are composed of both plastic and perfectly elastic components. This is clearly borne out by our experience with solid bodies that are stressed within their destruction limit. By sensitive measurements one observes an extended creeping deformation following a relatively large initial deformation. If the material is loaded so that the rate of deformation is sufficiently slow, then upon relieving the load the principal part of the deformation is eliminated at the speed of sound, whereas the deformation due to creep disappears at a very slow rate. When the deformation velocity is finite the material does not return to its initial state after being unloaded and a permanent deformation, corresponding to a dissipation of elastic energy, is thereby produced. A permanent deformation produced within the elastic limit of a material has been called by Joffe "an elastic after effect". Elastic hysteresis, damped vibrations and elastic fatigue are familiar phenomena caused by the elastic after effects that exist in materials. The importance of the finite rate of deformation in producing an elastic after effect in ordinary solids suggests that the phenomenon (mechanism) is quite similar to the mechanism for viscosity of a fluid.

It might be of interest to point out here that an elastic after effect is usually not detected experimentally both in single, metallic and non-metallic crystals. It is therefore a phenomenon which is in general determined by forces between atoms of distinct crystals in materials composed of many crystals. These forces

are therefore markedly distinguishable from the solidifying forces existing between atoms of a single crystal, as they permit permanent deformation within the elastic and destruction limits of the individual crystals composing the agglomeration. It is important to emphasize here that measurements show that the elastic after effects increase with the degree of atomic and molecular irregularities and complexities of a body. It appears that the forces associated with producing elastic after effects bear a striking resemblance to the Vander Waals forces in a liquid.

Other types of permanent plastic deformation have been observed in single crystals by various experimenters. These, however, do not occur within the elastic limit of the crystal and are associated with the removal of solidifying forces between some of the atoms of the crystal. The removal of these forces is apparently accomplished by the shearing stress and the thermal fluctuations of the heat bath containing the crystal.

A shearing stress distorts asymmetrically the potential barrier of atoms subjected to it (see Eyring (13)), thereby reducing the expectation time and increasing the virtual volume through which a thermal fluctuation of a given energy level will occur. The forces that exist between atoms which admit various kinds of permanent deformation described above are similar in at least one fundamental respect: namely, they are not solidifying forces. In this respect the various types of plastic deformation are

analogous to the deformation properties of liquids.

The atomic and molecular forces associated with permanent deformation may differ appreciably for different materials and for different types of deformation. For example, the forces acting between the molecules of an associated liquid are in general stronger and of a different type than those acting between the molecules of normal liquids which are of a non-directional character. The liquid state is characterized by two essential properties, which are cohesion and fluidity. These properties tend to act in opposite directions. A gas is distinguished from a liquid by the absence of cohesion between its molecules. As cohesion can exist in various degrees, the transition from the liquid to the gaseous state is not necessarily discontinuous. In a liquid state, as distinguished from a solid state, the atoms are not confined to fixed equilibrium (lattice) positions by solidifying forces but in time move throughout the entire region of the liquid. Nevertheless, in a liquid the molecules cannot be regarded as independent since cohesive forces exist between them. A theoretical approach to the liquid state is therefore more difficult than the relatively successful approaches that have been made in treating the solid and the gaseous states theoretically.

The plasticity of a material is also characterized by the existing cohesive forces between and the fluidity of some of

its atoms and molecules. In this sense it is again physically meaningful to identify the plasticity of the material with some properties which characterize the amorphous state in its most general aspects. The cohesive forces of liquids are distinguishable from the solidifying forces by the property that the latter do not permit the atoms to change their equilibrium position during or after deformation.

Eyring has developed an interesting "reaction rate" theory for viscosity and diffusion in a liquid (13) based upon the concept that a liquid is distinguishable from a solid by the presence of "holes" that permit the atoms to change their location as the result of thermal agitation and work done by the shearing stresses. From this theory it follows that the viscosity is entirely determined by the available number of "holes" contained within a given material and consequently the viscosity depends only upon the density of the material. In other words, changes in pressure and temperature, according to Eyring, should not produce a change in the viscosity of a fluid if the specific volume of the liquid remains constant during these changes.

The above conclusion is strikingly confirmed by Bridgman's measurements (14) in the case of non-associated fluids to which Eyring's analysis is virtually restricted. Although Bridgman's measurements show that at constant density changes in pressure and temperature produce changes in viscosity in other types of fluids, their effect upon viscosity is nevertheless small

compared to the effect of density. This conclusion is also indicated by a semi-empirical expression due to Andrade (15). Andrade's theory assumes that viscosity involves a temporary freezing together of molecules in a liquid into larger aggregates which are somewhat crystalline in character. According to Bridgman this freezing affect is essentially equivalent to the molecular interlocking mechanism that he proposed for explaining why the viscosity of a liquid depends so strongly upon the complexity of its molecules. The validity of this interlocking mechanism is apparently borne out by some of Bridgman's measurements.

As it has already been indicated, that the purpose of this part of the thesis is to determine whether, from our present knowledge of the actual structure and properties of matter, it is possible to infer some of the macroscopic properties of the materials situated in the interior of the earth, we now endeavour to relate the above considerations to the specific problem.

One can infer from the outline given above some results that are of geophysical interest, i.e., it seems possible to deduce from our present knowledge of the atomic structure and properties of matter some of the macroscopic properties of the materials situated in the interior of the earth. By knowing the macroscopic properties of these materials one can select suitable parameters for describing them mathematically in accordance with

physical principles. Accordingly, in the following, some inferences that are helpful for selecting these parameters are summarized:

a) The existence of solidifying (homopolar) forces has been established experimentally by the critical melting temperatures, latent heats of melting, and by the well-defined elastic limits found in a single crystal. Some of the most disturbing questions concerning the nature of solidifying forces and their stabilizing effects have been explained by F. London and G. Heitler (ibid) showing that a suitable linear combination of two unperturbed degenerate wave functions of a diatomic molecule admits a coupled (resonance) state which has a lower energy level (more stable) than any of the degenerate states. This important question, and more generally questions pertaining to lattice stability, had not been resolved by the Born (16) electrostatic theory of crystal lattices.

b) Liquids and gases are distinguished from solids by the absence of solidifying forces between some of their atoms. This does not imply that all forces between atoms of different solid components (molecules) of liquids are necessarily similar. However, they are similar in the sense that they are not solidifying forces and since these play a most fundamental role in the structure of matter, it is physically significant to classify matter according to whether or not these forces exist. It follows, therefore, that materials consisting of some atoms that

are solidified and others that are not, are composed of both solid and liquid components. The only materials that are known to behave as perfectly elastic below the elastic and destruction limits are single crystals. All polycrystalline materials are therefore, in general, composed (in different proportions) of both solid and liquid components, since such materials contain some atoms which are free of solidifying forces.

c) The materials of the earth, extending from the surface at least to depths at which deep-focus earthquakes are known to originate, most probably consist of agglomerations of both perfectly elastic (solid) components and what might be called amorphous components. The distribution and relative proportion of these contained in a material situated at a given depth in the earth depend primarily upon its chemistry, its principal and shearing stresses, and its temperature.

d) Since a body composed of both solid and liquid components is plastic, it follows that its size and shape (strain) are not single valued functions of the external forces (stress) but depend upon the entire history of loading and deformation. In the course of this history each stress has an influence which slowly dies away. In a particular state of deformation there exists traces of stresses that have been removed for a considerable time, but remants of which are still present. It follows therefore that in general, the strain on such materials depends upon time as well as stress and hence Hooke's Law cannot be used for the purpose of describing the deformation mathematically when they

are subjected to seismic disturbances. When the elastic components of a material are significantly dominant compared to its liquid components it is valid to use Hooke's Law as a first approximation. However, as far as the materials in the interior of the earth are concerned, using Hooke's Law is equivalent to begging the principal question at hand; that is, by assuming apriori that the effects of the liquid components are negligible, the analysis does not permit an aposteriori evaluation of the relative importance of the liquid components from the seismic data. In order to describe mathematically the propagation of waves in the interior of the earth with a generality that is sufficient for this purpose it is necessary to replace Hooke's Law by suitable laws which assert the dependence of strain upon stress and time. In doing so it seems necessary, for computational reasons, to restrict oneself to a physical continuum which is naturally limited to physical macroscopic phenomena. However, the stress, strain and time laws thusused should be guided as far as possible by available knowledge of the mechanisms of plastic deformation, and by test data obtained for various types of plastic materials undergoing deformation. It is also important to have some understanding of the effects of pressure, temperature and stress upon these mechanisms.

Part III of the thesis is concerned with some mechanisms of plastic deformation, plastic rupture and with test results revealing the nature of different types of plastic deformation

on a macroscopic scale. With this information it is then possible to approximately represent some of these characteristic types of plastic materials by differential equations relating stress, strain and time. Such a theoretical approach to the macroscopic aspects of plastic deformation of engineering materials under static loads has been used with some success by the Prager school of plasticity at Brown University, and others. In some cases, however, the stress-strain-time laws and the equations of equilibrium do not constitute a system of differential equations which determine all the components of the displacement vector as a function of the space coordinates and time. By introducing physically significant yield conditions into the analysis one can sometimes overcome this difficulty. Numerous yield conditions have been proposed in the literature (ibid. - Refs. (5), (6), (7), (8), (9)),with some giving consistently better results than others. The physical significance and value of a particular yield condition is also determined by the extent to whichit depicts the macroscopic aspect of the actual mechanisms of plastic deformation. It is apparent, therefore, that a satisfactory theoretical approach to plasticity based upon the continuum theory is effective only if it is guided by some understanding of the mechanism of plastic deformation on an atomic scale.

e) The fact that the materials in the interior of the earth are plastic rather than perfectly elastic leads to some

fundamental geophysical implications. One of the classical problems in geophysics is to explain the causes and mechanisms for periodically storing and releasing seismic energy in the earth, and the causes of diastrophism in general. If we evaluate this problem from a purely macroscopic point of view, we are restricted to call upon macroscopic changes in the earth as causes for the macroscopic effects we are trying to explain. This self-imposed restriction has usually led to contradictions with the second law of thermodynamics or else to difficulties in explaining the sources of energy.

In this regard consider a uniformly heated perfectly elastic body deformed by static loads. The only way the states of deformation of such a body can vary with time is by changing its external loads or its temperature. This however is not true of a plastic material. Even though its macroscopic environment is constant the plastic material can in general change its state of deformation in the course of time. This deformation is then obviously determined by atomic and molecular changes which connot be explained macroscopically. When we speak of a statically loaded body at constant temperature in the strictly phenomenological sense, it implies that the macroscopic state of the body cannot be altered by the physical phenomena represented by its temperature. However, if one considers the atomic and molecular processes which determine the macroscopic temperature of the body it follows that the stability of the bonds between the atoms of the body is a function of time, even

though its temperature is constant. The changes that are thus produced in the atomic bonds of the body cause it to change its state in the course of time when subjected to constant loads.

The temperature of a body represents the thermal energy imparted to it by a heat bath consisting of atoms and molecules that impinge upon the atoms and molecules of the body. The atoms of a material are in general enclosedby potential barriers. The transfer of energy between the atoms and molecules of a heat bath to the atoms of a material contained therein can in general cause at least some of the atoms to escape their potential barriers,and thus change the macroscopic qualities of the material. Saying that the macroscopic temperature of the body is constant is therefore equivalent to saying that the macroscopic temperature of its heat bath is constant. The macroscopic temperature of the heat bath is, however, the mean value of the temperatures of the individual components of the heat bath. The temperature of a single component is here interpreted as a measure of the kinetic energy corresponding to its random motion. Consequently, there are in general some components in the bath with thermal energies greater and less than the average thermal energy, although the thermal energy of most of the components are in the neighborhood of the macroscopic (mean) value. Even when the macroscopic temperature of the heat bath is such that all the potential energy barriers of the atoms of a material contained therein are

greater than the macroscopic energy of the bath, there always exist individual components in the bath with energies sufficiently large to free the atoms of the material by impinging upon them. The number of these components, and hence the probability of such interaction at a given energy level, increases with the macroscopic temperature of the body. At high temperatures the deformation of the body proceeds at a higher rate than at low temperatures. The time-dependent aspect of this type of deformation is determined by the components of the bath whose thermal energies are greater than the mean value.

The average time, t, required for the components in the heat bath of temperature, T, (absolute) to transfer to the atoms of a body the energy of their potential barriers, E, is given with good approximation by the equation $t = \lambda e^{\frac{E}{KT}}$ where k is the Boltzman constant, λ is a constant of the order of 10^{-13} seconds. The constant λ is a measure of the frequency in which the components of the bath impinge upon the atoms of the body. The expression $t = e^{\frac{E}{KT}}$ is a measure of the improbability for an energy of magnitude E to be imparted locally in the body by a heat bath having a mean temperature, T. What is interesting to note here is that the expectation time, t, is in general finite for all finite and constant absolute temperatures T, and that it decreases exponentially with T. The expectation time decreases very rapidly with the quantity E/KT; and for small values of this quantity the expectation time is very sensitive to small changes of either E, or KT.

It follows, therefore, that at high temperatures one should expect the materials of the earth to undergo significant changes and deformation even though their macroscopic environment, characterized by their external loads and temperatures, is unchanged. This type of deformation of the materials of the earth, as well as its periodic aspect, is consistent with and in a sense is a counterpart of the second law of thermodynamics. It is emphasized that although a heat bath can be an energy source for this type of deformation, the existence and maintenance of temperature gradients in the earth are not necessary to cause the types of deformation considered here.

When a material is subjected to loads producing shearing stresses, the potential barriers of its atoms are lowered on one side and raised on the opposite side. In other words, a shearing stress has the effect of assisting the atoms to escape on one side of their barriers and of resisting their escape on the opposite side. The change in shape of the potential barriers so produced, reduces the expectation time required for escape on the lowered side and increases it on the elevated side, thus producing a time-dependent shearing deformation.

A shearing stress can displace atoms without changing the average distance between them. If we postulate that the solidifying forces, or more generally, that the forces necessary for maintaining a crystal lattice are functions only of the distance between atoms, it follows that the magnitude of these forces is not sufficiently changed by a shearing deformation. Consequently,

the changes in shape and amplitude of the potential barriers associated with the shearing deformation are primarily determined by the shearing stresses and only slightly affected by the small changes of the inter-atomic forces.

A material subjected to hydrostatic pressure would therefore behave differently in this respect, since the average distance between its atoms is in general decreased by the application of pressure. Hence, in a deformation produced under hydrostatic pressure as distinguished from a shearing deformation, the interatomic forces change and thus modify the potential barriers of the atom. Bridgman found that the compressibility of most materials tested decreases with pressure although in a few exceptional cases the compressibility was found to increase with pressure. This implies that, with few exceptions, pressure has the effect of steepening the walls of the potential wells and thus increasing the resistance of the material to further compression. It is plausible to assume that the work done by hydrostatic pressure will increase the amplitude of the potential barriers symmetrically and thereby reduce the probability of atoms to escape as a result of thermal fluctuations. These considerations would explain why materials do not yield plastically to hydrostatic loads producing stresses much greater in magnitude than the yield stress of the same materials in shear. Bridgman's experiments indicate, however, that at very high pressures the compressibility of some materials has directional properties, i.e., the deformation is non-isotropic. This phenomenon may be attributed to the effect of close packing of atoms and molecules with asymmetric force fields of interaction. This would also explain why polymorphic

transitions of crystals, caused by high pressures. usually lead to a reduction in their symmetry. At relatively low pressures, however, the atoms are not as closely packed so that the field of force of each atom which determines the lattice structure tends to approach more closely spherical symmetry. It is thus experimentally established that very high pressures, as distinguished from low and moderate pressures, can increase the asymmetry of the potential barriers of the individual atoms of solid material.

It has also been established experimentally that sufficiently high pressures can cause the atoms of a solid to escape their equilibrium positions and rearrange themselves in a new set of equilibrium positions, thereby changing the lattice symmetry of the material. Although a quantitative theory for the analysis of time rates of polymorphic transitions is apparently not available, it is nevertheless possible to infer from the experimental results obtained by Bridgman (ibid.) some significant geophysical implications of polymorphism. Accordingly it has been found that the temperature coefficient for the velocity of some polymorphic transitions is very large; which means that the rate of transition in such cases is very sensitive to temperature, and can therefore be considered as a rate process. it should thus be expected that under very high pressures polymorphic transitions of a material can be induced by thermal

fluctuations of its heat bath. Thus when the potential barriers are sufficiently distorted by high pressure, the thermal fluctuations work with rather than against the pressure to produce polymorphic transitions in a manner which is similar to the effect of thermal fluctuations upon a shearing deformation.

If and when the potential barriers become sufficiently distorted by the pressure, thermal fluctuations can assist the atoms to escape their barriers and relocate themselves in a new set of equilibrium positions thus producing polymorphic transitions. Consequently it is probable that in the interior of the earth, where high temperatures and pressures prevail simultaneously, the mechanical equilibrium of the materials situated there can be suddenly disturbed by polymorphic transitions. Therefore, as in the case of a shearing deformation, time changes of state and mechanical equilibrium associated with polymorphic transitions do not require time changes in the macroscopic environment of the materials.

Although qualitative, the above inferences based upon an interpretation of plasticity, polymorphism, etc., as reaction rate processes are not speculative from a modern physical point of view. Instead they follow from accepted principles of statistical and quantum mechanics. That is, in principle the fluctuations in a heat bath with a sufficient high mean temperature, T, can produce the energy required to cause quantum transitions in a material. Since there is no apparent physical reason for

excluding such phenomena in the interior of the earth; and since the temperatures at great depths are probably very high, these phenomena are probably quite prevalent.

In this part of the thesis an attempt was made to establish the importance of rate processes as a fundamental cause for chronological changes in mechanical equilibrium of the earth without considering specific mechanisms for producing them. To do so it is convenient to summarize and interpret from a geophysical point of view some available knowledge pertaining to the physics of plasticity. This is covered in Part III of the thesis.

III. Plastic Deformation and Plastic Rupture:

A. Plastic Deformation of Single Crystals:

In Part II of the thesis it is concluded that the materials of the earth are essentially plastic to a greater or lesser degree. Some general, although apparently fundamental, geophysical implications of these conclusions were presented.

In attempting to establish some specific causes for seismic activity determined by the plasticity of the materials in the earth it is helpful to examine first the physics and mechanisms for various types of plastic deformations. For this purpose we shall consider mechanisms for plastic deformation determined empirically; as well as some characteristic plastic properties of various materials determined by laboratory tests conducted on a macroscopic scale. The mechanisms for plastic deformation of polycrystalline materials and their geophysical implications can be clarified and somewhat systematically determined from experiments with single crystals.

A. Joffe (ibid.) and his school have determined two principal mechanisms for plastic deformation in single crystals by using polarized light and X-ray diffraction techniques. An investigation was initiated to determine whether or not elastic after-effects, elastic hysterisis, and elastic fatigue of polycrystalline materials, can be measured in single crystals.

It was found that these phenomena were all absent in non-metallic and metallic crystals tested when stressed below their elastic limit.* These results imply therefore that elastic after-effects and related phenomena observed in polycrystalline materials, within their elastic limits, must be attributed to changes occurring between atoms within a single crystal.

When a single rock salt crystal was subjected to a uniformly distributed tensile load it was observed that the X-ray pattern of its lattice remained unchanged. The load was gradually increased until it reached a critical value at which point the pattern changed abruptly. Instead of one set of spots, corresponding to the reflections from the lattice planes of the crystal, a number of such sets appeared corresponding to lattice planes of several crystals. All of the new spots, with the exception of one, were distributed about the lattice reflections of the mother crystal. This implies that the mother crystal and the newly formed crystals have one lattice plane in common. According to Joffe, the newly formed crystals are displaced in the (110) plane and rotated about an axis perpendicular to the plane, corresponding to the unchanged spot of the diffraction pattern. The twinning and kinking (13) of individual crystals can be associated

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*It is noted that plastic creep in single crystals is a phenomenon which occurs at and above rather than below the elastic limit.

with the abrupt change of the crystal lattice at the destruction limit. The abrupt change in the X-ray pattern corresponds to an irreversible change of the crystal lattice and is called by Joffe "the destruction limit".

The extremely critical nature of the destruction limit suggests that it is caused by the abrupt removal or modification of solidifying forces between some of the atoms or ions of the mother crystal. This explanation is supported by other experiments which show that the destruction limit decreases rapidly with temperature and is zero (stress) at the melting point of the crystal. Since the melting point corresponds to an abrupt removal of the solidifying forces in a solid, and the destruction limit is zero at this point, it is indicated that the mechanisms for fusion and destruction at lower temperatures are very similar. It was further observed that the destruction limit depends primarily upon the internal stresses and is virtually independent of the size of the crystal. It was also found that in general the rupture of a plastically deformed crystal requires higher tension than does a single crystal which is not plastically deformed.

According to Joffe, this implies that the distance between atoms of different crystals formed at the destruction limit is not greater than the distance between atoms in the mother crystal. It should be noted, however, that the increase of the rupture

strength of a crystal beyond its destruction limit does not necessarily imply that the potential barriers of all the atoms in the fragmented specimen are similar or that solidifying forces exist between all its atoms. In fact, the absence of an elastic after-effect in the mother crystal and its presence beyond the destruction limit is conclusive proof that at the destruction limit there exist some atoms in the specimen which are free of solidifying forces.

It will be shown later that plastic deformation, associated with the destruction limit of a single crystal, leads to some significant geophysical implications. These result from the strong dependence of the destruction limit upon temperature.

Another mechanism (type) of plastic deofrmation that exists in a single crystal is associated with its elastic limit and is caused by slip of crystalline sheets parallel to a crystallographic direction, without rotation. The permanent set caused by pure slip of the crystalline sheets does not produce a change of the X-ray diffraction pattern and hence is not detectable by such a technique. However, using polarized light and watching the crystal between michols gives a very sensitive technique for determining the load which produces the first permanent set (plastic deformation) of a single crystal. This type of permanent set first appears when the load is about 10% of the destruction load and is indicated by the appearance of a sharp, bright line on an otherwise dark background. This line does not appear when the load is removed and consequently corresponds to a permanent deformation. The first permanent set produced defines physically

the elastic limit of the crystal. When the crystal is stressed by a constant load slightly beyond its elastic limit, newlines parallel to the first appear in succession. In the case of rock salt these represent projections of the (110) planes of the crystal. The time interval between the formation of successive lines increases until finally new lines do not appear. At this point elastic equilibrium is again established at a stress greater than the one corresponding to the initial elastic limit. It is necessary to increase the load further in order to produce new lines corresponding to the development of new slip planes. The additional load thus required increases with the number of slip planes that have already been formed.

Measurements by M. W. Klassen (17) have determined that the shear deformation due to s lip in a heated rock salt crystal progresses in jumps. For a constant load the time interval between jumps increases until finally the jumps are discontinued. The frequency of the jumps increases with load and becomes very small for loads that are only slightly greater than the load required for yield. The amplitude of the jumps increase with temperature and appear to be independent of load. These experiments thus indicate that at low temperatures and high loads (shearing stresses) this type of plastic deformation would tend to be relatively continuous; whereas in the deformations produced at high temperatures and for shearing stresses slightly greater than critical, the jumps would become accentuated.

These results can be simply explained. When the applied

stress is only a little larger than the critical stress (required to produce yield) the jumps in the shearing deformation which occur as a function of time, are primarily caused by thermal fluctuations. However, when the shearing stresses are sufficiently large they can cause plastic deformation with little assistance from thermal fluctuations and the deformation would thus appear relatively continuous.

At very large depth in the earth where the difference between the principal stresses and hence the shearing stresses can be expected to be small, and the temperature is high, plastic deformation due to glide can be expected to occur in relatively large jumps at large time intervals. Accordingly, a mechanism for seismic activity based upon plastic yield is thus indicated.

Since thermal fluctuations play a fundamental role in plastic glide and since the yield point depends strongly upon the temperature it appears probable that at sufficiently high temperatures, yield can be produced by thermal fluctuations when the applied shearing stress is less than the critical (yield) stress. It is noted that although some important questions concerning the details of the creep mechanism are still open, available experimental results nevertheless clearly establish that temperature and therefore thermal fluctuations play a fundamental role in yield and creep phenomena. It is thus indicated that creep is sensitive and it is to a large degree determined by atomic phenomena, not revealed by X-ray diffraction.

Creep is in part determined by a large class of "sensitive" phenomena known to exist in single crystals.

Among these properties of a crystal are hardness, electrical conductivity, thermal conductivity and rupture strength. The sensitive properties of a crystal are not determined by the atomic structure of the unit cell of the lattice but are apparently determined by properties that are not fully developed in the unit cell. The existence of "structure sensitive" properties in single crystals suggest that they originate from nuclei. This concept has been used by Orowan (18) in the development of a satisfactory theoretical explanation for plastic rupture based upon a machanism for crack propagation. As a result the "notch effect" concept of A. Griffith (19) requiring comparatively large cracks to explain the low rupture strength of plastic crystals, becomes more plausible. The principal difficulty encountered by the Griffith theory is thus removed by the Orowan theory, which calls upon the existence of sensitive properties in plastic crystals. It appears that "structure sensitive" phenomena are connected with an unstable arrangement of atoms whose equilibrium can be disturbed by localized disturbances produced by thermal fluctuations. These phenomena can therefore be called upon to provide a "trigger mechanism" for effecting changes of state in materials which are produced by thermal fluctuations. In particular, they can play such a role in the production of seismic disturbances and thereby reduce the expectation time required for producing them in a heat bath.

It has been previously noted that the particular mechanism for plastic deformation of a crystal which is related to its elastic limit has the effect of increasing the yield stress. The mechanism associated with the destruction limit of a crystal does not have the effect of increasing the stress required to cause further fragmentation of individual crystals. This follows since it has been shown experimentally (20) that the destruction limit (stress) of a single crystal is independent of its size and time for temperatures ranging between 20° C - 600° C. Since the destruction limit is very sensitive to temperature it is probable that at very high temperatures it is a function of time, especially when the crystals are small and when the destruction depends upon "sensitive" properties of the lattice. This is indicated by interpretations on a molecular scale. Accordingly, it follows from the Polanyi-Wigner relation (21) that the expectation time for a quantity of heat energy to be localized in a heat bath is in general finite. As the mean temperature of the bath increases the expectation time required for the bath to locally (within a given region) produce a given quantity of heat energy decreases exponentially. It would therefore be desirable to conduct some crucial experiments at high temperatures and of extended duration, in order to check this conclusion. That is, it is desirable to determine whether or not the destruction limit at very high temperatures and pressures is a function of time as well as stress. That such a phenomenon has not been observed within the temperature range covered by Joffe's experiments may be due to the possibility that the expectation time required to cause fragmentation with (applied)

stresses less than critical exceeded the duration of his experiments.

In this thesis thermal fluctuations have been and will be called upon to explain disturbances in the mechanical equilibrium of the earth materials in the absence of temperature and other macroscopic gradients. It is therefore, relevant to emphasize that such an explanation for seismic disturbances is not speculative from a physical point of view but rather stems from modern physical principles. This follows from physical consideration leading to a result which can be conveniently summarized here in the following (theorem) statement: - The principles of statistical mechanics imply that all changes of state and phenomena of materials that are sensitive to changes in mean temperature of their heat bath are in principle also sensitive to thermal fluctuations of the heat bath.

The probable expectation time required for a thermal fluctuation of given intensity to occur in a given volume of material decreases with the mean temperature of the heat bath. The probable expectation time required to experience a thermal fluctuation extending throughout a volume of material, increases with the volume. Consequently, changes of state restricted to small volumes of a material can be produced by a fluctuation of given intensity more frequently than corresponding changes extending over a larger region of the material. If we take into

consideration the "structurally sensitive" phenomena originating at nuclei in a material, it follows that the expectation time required to produce these phenomena is less than would be necessary for producing the same changes if the materials are "structurally insensitive". The existence of sensitive phenomena in crystals reduces the expectation time and increases the effective spacial extent of a thermal fluctuation of a heat bath. Since these conclusions are general they apply to both types of plasti c deformation in individual crystals described above as well as to plastic rupture considered in the next section.

A collection of important papers relating to "sensitive phenomena" in crystalline materials can be found in reference (22). Among these, reference is especially made to the papers by E. Orowan, A. Smekal and A. Joffe.

B. Plastic Rupture:

W. C. Burgers (23) has found experimentally that shear hardening of a material due to plastic deformation is significantly affected by the "sensitive properties" of the material. The effect of these properties of a material upon its plastic rupture strength is, however, much more critical and pronounced. It has been determined experimentally that the rupture strength of a large number of materials such as rock salt, mica, glass, etc., are generally much lower than the calculated theoretical values. It has been consistently observed by different investigators, however, that the rupture strength can be increased by a hundred times and more by controlling surface and loading conditions. Thus, the measured values of rupture strength

can be made to approach the theoretical values. These results imply that the reduction in strength of a crystal can depend upon physical conditions that are not represented by the lattice structure observed by X-rays and must therefore be attributed to "structure sensitive" properties of the crystal. From previous considerations it would therefore follow that thermal fluctuations may be expected to produce plastic rupture under suitable conditions. Plastic rupture so produced may therefore be called upon as another principal source of seismic energy originating in the interior of the earth.

The "notch effect" mechanism due to Griffith (24) and the mechanism of plastic crack propagation due to Orowan, apply directly only to a single crystal grain in a polycrystalline material. The plastic rupture of a polycrystalline body in general requires a larger shearing stress than does a single crystal. Due to the different lattice orientations of distinct grains, the propagation of a crack formed in a single crystal across its boundary requires a larger stress than necessary for the crack to be propagated (in the sense of Orowan) within the grain itself. This explains why polycrystalline bodies are stronger than the single crystalline grains of which they are composed, as well as the strengthening effect of foreign bodies contained in a polycrystalline material.

C. The Plastic Deformation of Polycrystalline Materials:

The plastic deformation of polycrystalline materials is governed by the mechanisms of plastic deformation of single crystals as well as by forces between crystalline grains. The existence

of elastic after-effects and elastic hysteresis in these materials conclusively establishes the fact that some of the intergranular forces are similar to those existing between molecules of an amorphous (liquid) material.

The effects of thermal fluctuations in producing sudden time changes in the state of plastic crystals and their geophysical implications, presented in section III-A are therefore also effective in polycrystalline materials. In addition, however, thermal fluctuations impinge upon atoms subjected to intergranular forces and thereby change their locations within the material. These changesproduce a plastic deformation, typical of amorphous materials. In general the amorphous properties of a polycrystalline material can be attributed to the presence of grain boundaries as well as any other structural disorder. Plastic deformation caused by thermal fluctuations in an amorphous material is essentially continuous and therefore cannot in general be called upon to explain sudden seismic disturbances.

D. Plastic Deformation of Amorphous Materials:

The molecules of an amorphous material do not have a regular geometrical arrangement as do the molecules, atoms or ions in a crystal. This is equivalent to saying that the probability function representing their spacial distribution does not have sharp maxima indicating that some locations are much more probable than others. Such maxima exist in a solid (crystalline) material, corresponding to the lattice points and equilibrium positions determined by the solidifying forces.

The forces between molecules of different amorphous materials, in general differ. In some materials they are much stronger than others. Bridgman (25) has found that these forces can be significantly increased with pressure and that viscosity and other properties of amorphous materials can differ markedly from those of a Newtonian fluid.

The molecules of an amorphous material can be rearranged by thermal fluctuations. Since they can move more or less independently, (depending upon the magnitudes of the intermolecular forces) the expectation time requiredto relocate a molecule is much smaller than required, to produce fluctuation sufficiently extended and intense to cause plastic deformation of a crystal involving many atoms. In the absence of external loads the rearrangement of molecules in an amorphous body is random and consequently does not result in a macroscopic deformation. The application of an external load distorts the relatively weak potential barriers of the molecules so that the probability of thermal fluctuations producing a deformation in the directions of the external stress is the maximum. This causes a continuously increasing deformation in the course of time and is therefore, in the large sense of the term, a plastic flow. If the mutual energies of the molecules are only slightly affected by the external stress then the thermal fluctuations are the principal energy source for rearranging the molecules in a direction determined by the external stress. Orowan (26) has shown by expanding the mutual energy between molecules as a power series of the applied stress that when only the linear term

is retained the result is a flow, the rate of which is, proportional to the stress, i.e., Newtonian viscosity. When the mutual energy is strongly affected by the applied stress or by the strain history of the amorphous material, then higher order terms in the expansion must be retained and thereby represent an almost continuous transition from non-Newtonian viscosity to crystalline plasticity. This should be expected since the solidifying forces are precisely those which are determined by the mutual energy between atoms and molecules.

In a gas, as distinguished from a liquid, intermolecular forces are absent and its Newtonian viscosity does not depend upon a thermal activation mechanism, but is completely determined bya momentum transfer mechanism.

For further consideration of the macroscopic plastic properties of polycrystalline and amorphous materials it is convenient to consider typical stress, strain and time relations for such materials.

E. The Description of Macroscopically Observable Plasticity by Typical Stress, Strain and Time Relations:

Usually the stress, strain and time relations for various plastic materials are experimentally determined by measuring strain as a function of time for a constant stress. The strain-time relations so obtained, are essentially two types, characterized by a terminating strain in one case and a non-terminating strain in the other case. When a stress is applied to a strain terminating

material the rate of plastic strain is a maximum at the beginning and approaches zero in a finite time, the strain thus terminating to a finite value. When for a constant stress the dependence of strain upon time is governed by a system of linear differential equations the strain-time relation is exponential, and the time required for the strain to reach (1 - 1/e) of its terminating value is usually called the "orientation" time Upon the removal of stress in such a case, the strain will usually decrease exponentially and the time required for it to decrease 1/e times its initial value will, in general, be equal to the orientation time defined above.

In the case of materials with non-terminating strain properties the strain does not terminate to a finite value but continues to increase indefinitely. The strain in this case can generally be divided into two parts, the first is terminating and recoverable upon removal of the stress, whereas the second part is non-terminating and is not recoverable. The recoverable and non-recoverable deformations are known as primary and secondary creep, respectively.

In examining the plastic properties of some materials, it is sometimes convenient to maintain the stress constant and observe how the strain decreases with time. When the relation between stress and time so obtained is exponential, the time required for the stress to decrease 1/e times its initial value is called the "relaxation" time.

It is noted that in general the differential equations which govern strain as a function of time are not necessarily linear, in which case the dependence of strain upon time is not necessarily given by a sum of exponential functions of time. The stress required to maintain a constant state of strain in a strain terminating material in general decreases with time, although not necessarily exponentially. When the stress in such a case is relaxed exponentially with time, the time required for the strain to decrease by 1/e times its initial value is called the "relaxation" time of the material. Non-linear processes of plastic deformation can in some cases be considered linear to a first approximation. For example, such an approximation may be applicable to small deformations and for short periods of time, even though the dependence of strain upon time is non-linear. When the plasticity phenomena are linear, relaxation and orientation as defined above should be given by a sum of exponential time functions. In these cases, therefore, the phenomena can be represented by spring dashpot combinations which do not necessarily have physical significance.

If plasticity is considered as activated or thermally-excited process, it follows that linear phenomena such as exponential orientation and relaxation can occur only when the energy differences are small compared to the thermal energy (KT). This can be seen by inspecting the Maxwell-Boltzmann probability distribution function $e^{-W/KT}$ which is linear in W only when

W/KT is small, namely $KT \gg W$.

The concept of Temporal probability or transition rate is simple and therefore useful quantitatively only when the interaction field between the elements (atoms, molecules, etc.) is not significantly modified in the course of time by the changes produced by the thermal fluctuations, i.e., when the thermal fluctuations do not change the nature and average distribution of forces between the elements. In order that significant changes in the interaction field produce linear effects on a macroscopic scale it is necessary for them to be dispersed. Their effects upon modifying the interaction field thus cancel statistically. This is much more characteristic of fluids than it is of solids, therefore, when exponential strain-time relations are measured in the case of large strains it suggests that the material involved behaves essentially as a liquid (27).

Plastic glide usually changes the environment of the interaction field and can thus introduce important non-linear effects. These changes are not usually dispersed as are, for example, lattice injuries.

The above considerations embody the following geophysical implications:

1. Significant information relating to the nature of the plasticity of seismically active materials can be inferred from the function obtained from measured strain-time increments associated with earthquakes. Thus a linear (differential) strain-time relation would imply in general that the changes in the interaction field leading to plastic deformation are dispersed at

random whereas a non-linear relation would imply that changes in the interaction field are not dispersed at random.

2. The probability of random dispersion increases as the volume of the material under consideration is increased. Thus it is natural to expect the differential strain-time measurements obtained over an extended region to be linear even though it may actually be non-linear when based upon a comparatively limited region. In such cases the linearity would be determined by the extent of the region from which strain-time data are obtained and would therefore disguise whatever non-linear characteristics may actually exist. For example, a Newtonian viscous fluid would naturally reveal an exponential strain-time curve, for a region of arbitrary dimensions, which is not generally true for other types of plastic materials. In the latter, case a linear relationship can be synthetically obtained by arbitrarily extending the dimensions of the region from which data contributing to such a relationship is selected.

3. The questions raised in (2) clearly establish the need of a criterion for selecting the dimensions of a region contributing data to an empirical strain-time function. By progressively reducing the dimension of the region contributing data, it should be possible to expose real non-linear strain-time characteristics, should they actually exist. In so doing it may be possible to locallize non-Newtonian plastic deformations in the earth and to refine its correlation with seismic activity Reference (1).

F. Analysis of Relaxation and Orientation Times from Experimental Data:

Exponential Analysis: Numerical and graphical methods for expressing an empirically determined time function for stress or strain in terms of a finite number of exponential functions are available, and are suitable when the phenomena are linear and can be described by a mixture of a few components (spring dashpots, etc.). Among these are methods developed by J. Whitehead (28) Leaderman (29), Levi (30), and Pierce (31). Each of these methods give one or more relaxation or orientation time constants associated with each exponential function. The reaction rate theory of Eyring relates the relaxation time so determined are to the free energies of activation in the material.

When the plasticity is linear but cannot be represented by a small number of exponential terms one can use a method based upon the Laplace Transform. References to such a treatment can be found in Whitehead's paper referred to above. This method is also preferable when one is interested in determining particular time constants of exponential functions which govern the phenomena over a restricted time interval.

Non-linear phenomena can sometimes be treated by a power law relation, developed by Scott-Blair (32). It has been shown that a variation of strain with time according to a power law can arise from a linear system only as an approximation over a small range of time, to which an exponential analysis is applicable.

If we reconsider the mechanism of plastic glide of a single crystal described earlier in the thesis, it becomes apparent that it embodies the strain-terminating, stress-relaxing properties observed macroscopically in polycrystalline materials. It therefore appears reasonable to attribute this kind of plasticity to glide deformations of individual crystals, in a polycrystalline material. If and when a non-terminating plasticity component exists in a material, it can be attributed to its amorphous rather than crystalline structure. The atoms, or more generally, the units, of the lattice situated near the surfaces of crystalline grains, as well as lattice injuries within the grains caused by plastic deformation, contribute to the amorphous properties of a polycrystalline material.

Changes of state of amorphous materials produced by thermal fluctuations are usually continuous. Brittle rupture of such materials when they are suddenly subjected to a load must be attributed to a lack rather than a surplus of thermal agitation. That is, the thermal activation is insufficient to permit the molecules to relocate themselves with a rapidity sufficient to prevent rupture. It follows, therefore, that in this case we cannot call upon thermal fluctuations to produce sudden changes necessary to cause seismic disturbances. In this respect, materials with a lattice structure (anisotropic bodies) are fundamentally different. The change of symmetry in the lattice must be discontinuous, since a symmetry element is either present or absent.

G. Statistical-Mechanical Considerations and Thermal Fluctuations:

Statistical thermodynamics distinguishes between two modes of transition which result in a change in the lattice symmetry of a material. In the first the probability distribution function for the equilibrium positions of the atoms changes simultaneously and abruptly with the change in symmetry; and at the transition point there exist two bodies in different states, which are in equilibrium with each other according to the Clapeyron-Clausium relation. Thus, if a thermal fluctuation of sufficient intensity and extent is produced and is not situated on the phase equilibrium curve, a sudden change of state will occur in the material. According to the Le Chatelier-Braun principle the system will act to reduce the effect that the thermal fluctuation has upon taking it out of its equilibrium position, and thus tends to restore its equilibrium. Thermal fluctuations which fall on the phase equilibrium curve can cause a local phase transition without disturbing the mechanical equilibrium of the material. Fluctuations which are not so restricted would probably disturb its mechanical equilibrium.

The second kind of transition also results in an abrupt change in lattice symmetry although the probability distribution function of the lattice elements (atoms, ions, molecules) can vary continuously. This type of transition is called the "Curie point transition". The continuous change in the probability distribution function corresponds to an incompletely ordered crystal lattice caused by the dislocation of

some of its elements without adding or removing any symmetry elements of the lattice. The change in symmetry caused by the incompressible plastic deformation of a single crystal (observed by Joffe) would appear to be of the Curie type. On the other hand, polymorphic transitions caused by high pressures are probably of the phase transition type. In both cases, however, it should in principle be possible for thermal fluctuations to produce phase transitions which do not correspond to points on the equilibrium curves and thus disturb the mechanical equilibrium of the materials involved. It shouldbe noted, therefore, that the changes of state which are proposed in this thesis as sources of seismic activity are in general consistent with the principles of statistical thermodynamics.

IV. On Seismic Mechanisms:

In Part III it was shown that plastic deformation of crystalline and amorphous materials can be interpreted as rate processes. It was also indicated that polymorphic transitions can, in principle, also be caused by thermal fluctuations. With the exception of amorphous placticity, it is thus reasonable to assume that the above processes can under suitable conditions occur abruptly and thus cause seismic activity in the absence of macroscopic gradients in temperature etc. Whether or not these abrupt transitions actually occur in the interior of the earth appears then to be a quantitative rather than qualitative question depending upon the temperature, state of stress and the chemistry of the materials. These quantities can be measured only indirectly with the help of suitable theories relating them to observable quantities. As has already been indicated, the development of analytical techniques suitable for this purpose apparently requires a Heuristic point of view. Laboratory tests can also provide useful information. However, the practical difficulties connected with such experiments which are conducted simultaneously at high temperatures and high pressures imposes practical limitations upon this source of information.

A. A seismic Mechanism for Shallow and Moderate Depth Earthquakes.

We will now consider in detail a specific mechanism for producing and repeating extended seismic disturbances at shallow and moderate depths.

In order to illustrate this mechanism consider a layer of essentially elastic material with finite thickness resting upon a plastic, strain-terminating stress-relaxing material. It is assumed that the displacement and stresses at the interface separating the two layers are continuous. Consider a shearing stress gradually applied to the elastic layer increasing in magnitude as time progresses. If the modulus of rigidity of the elastic material is much greater than it is for the plastic layer, little stress is thus transferred to the plastic layer until the elastic material either suddenly yields plastically or undergoes a sudden acceleration upon overcoming the static friction of a fault contained therein. When this happens a seismic disturbance is produced in the elastic layer at which time it experiences a comparatively large displacement and acceleration. By such a displacement significant stresses are transmitted to the plastic material as it is strained in accordance with the continuity conditions at the interface. Static equilibrium is then established between the external load applied to the elastic layer and the stresses distributed throughout the plastic and elastic layers. Thus a constant state of strain within the plastic material tends to be maintained in the course of time. Under such conditions, however, the plastic material proceeds to relax its stresses by thermal fluctuations;

resulting in a gradual increase in the shearing stress in the elastic layer so that the external load is balanced. The stresses so increased cause the cycle of the mechanism, outlined above, to repeat itself. This mechanism is based on the idea that the strain energy in an essentially elastic material is gradually increased to a critical value, at which it must accelerate (earthquake) to accommodate the applied load. The gradual increase of the strain in the elastic material is attributed to the relaxation of stresses in an adjoining plastic medium. Such a phenomenon would not occur and repeat itself if the shearing load is directly applied to a single plastic layer. Under such conditions a critical state leading to sudden acceleration (such as exceeding the elastic limit of a material or overcoming static friction along a fault) could not arise.

In practice, the shear loads applied to the upper layer can be attributed to gravitational forces produced by large masses, such as mountain ranges. In each cycle energy is dissipated and therefore the stresses in the upper layer are not completely recoverable. The energy thus dissipated requires a leveling-off of the potential energy of the masses giving rise to the external loads. The time required for this mechanism to cease its seismic activity would be determined by the initial load, the yield stress (or the stress required to overcome static friction of a fault) and by the physical constants of the plastic and elastic materials.

Since it appears at present virtually impossible to directly verify this mechanism from available empirical data at attempt is made in Part V to determine some quantitative implications of the mechanism which can be checked experimentally. In so doing the plastic sub-layer is represented by a Maxwell material, which is the simplest type of material embodied with stress relaxing properties. For this reason the physical configuration treated mathematically in Part V is probably an over-idealized system which may not admit solutions representing some surface waves that occur in practice. Such difficulties are actually encountered when accepted values for the viscosity coefficient are assigned to the appropriate material constants of the Maxwell material. It is felt however, that these difficulties are not essential and can be removed by replacing the Maxwell material with a less ideal plastic material. Thus for example a material consisting of a viscous element in parallel as well as in series with an elastic element is more representative of an actual stress relaxing plastic substance, and would probably remove the above mentioned difficulties.

It is noted however, that using a viscosity coefficient of the order of 10^{20} poises and rigidity coefficient of the order of 10^{10} dynes for the Maxwell material gives stress relaxation times which compare very favorably with the order of magnitudes of periods between successive earthquakes.

This fact alone provides fundamental empirical support for this mechanism which would not be significantly weakened by replacing the Maxwell material with a less ideal plastic substance, in the manner described above.

This mechanism applies to extended deformations which would be expected to occur primarily at shallow and moderate depths in the earth. In principle, the same mechanism would apply to two or more layers consisting of plastic materials having different material constants or properties. This mechanism is also consistent with and could explain the large fault displacements (horizontal and vertical) associated with shallow and moderate-depth earthquakes. This follows since a fault displacement would occur when the stress in the faulted layer is sufficiently increased, as a result of stress relaxation in the adjacent layer, to overcome the static friction of the fault. This mechanism also explains a method for storing energy required by the elastic rebound theory.

B. Deep Focus Seismic Disturbances.

Deep focus seismic disturbances may be attributed to plastic yield, plastic fragmentation of single crystals, plastic rupture and polymorphism, which are excited by thermal fluctuations. Disturbances originating in this way are most

probably confined rather than dispersed over extended areas. Such disturbances should be expected where the temperature is very high, i.e., deep focus. They may also be expected to occur at shallow and moderate depths if the temperatures are sufficiently high. In fact, earthquakes which show some correlation with volcanic activity might be attributed to one of these causes, and especially to polymorphism.

The probability of thermal fluctuations producing transitions from states of metastable equilibrium (which are thermodynamically possible) to stable equilibrium states is relatively large, in comparison to the probability of thus producing transition between two stable equilibrium states.

The possible causes proposed here for producing earthquakes at great depths would, from previous considerations, suggest non-linear relations for the macroscopic strain (or equivalent) resulting therefrom, as a function of time. Consequently, an exponential analysis of such empirical data is most likely unsatisfactory and a power relation is more suitable for large time intervals or large strains. On the other hand, the strain-time relations associated with the mechanism proposed here for moderate and shallow earthquakes are most likely linear and, if so, the data thus obtained can be analysed by the methods for exponential analysis referred to above.

V. On the Propagation of Love Waves in Plastic and Visco-Elastic Media:

Introduction:

The seismic mechanism described in part IV-A of the thesis does not readily lend itself to mathematical formulation or to direct experimental verification. It is therefore desirable to consider mathematically some seismic implications of the mechanism which may be verified by available data. As the selectivity in the period of Love Waves is not explained by theories restricted to elastic media, it is of interest to investigate the effect of a plastic sub-layer upon the propagation of such waves. In so doing it is found that shear waves can be propagated in the plastic material and that the observed restricted range in the period of Love Waves can be attributed to the presence of a plastic sub-layer. The mechanism proposed in part IV-A is based upon the existence of a plastic sub-layer consisting of a material with strain terminating and stress relaxing properties. In the following the plastic sub-layer is represented by a Maxwell material which embodies this stress relaxing property and is defined by a comparatively simple stress, strain and time law. In so doing it is tacitly assumed that the plastic state of the sub-layer is determined by the static stresses produced by the body forces of the upper layer and by the temperature of the plastic material. Thus it is assumed that the propagation of a seismic disturbance in the sub-layer does not modify the stress-strain-time law of the material. Otherwise it becomes necessary

to use different laws for the plastic state of the material when it is successively loaded and unloaded by a seismic wave. Thus the analytical study of wave propagation in plastic materials situated in the interior of the earth leads to mathematical considerations which are more tractable than those encountered in the study of plastic waves produced in initially elastic material, subjected to impact loads causing plastic deformation.

Analysis:

The configuration considered in the following analysis and the symbols used for the material constants are illustrated and defined by figure 1.

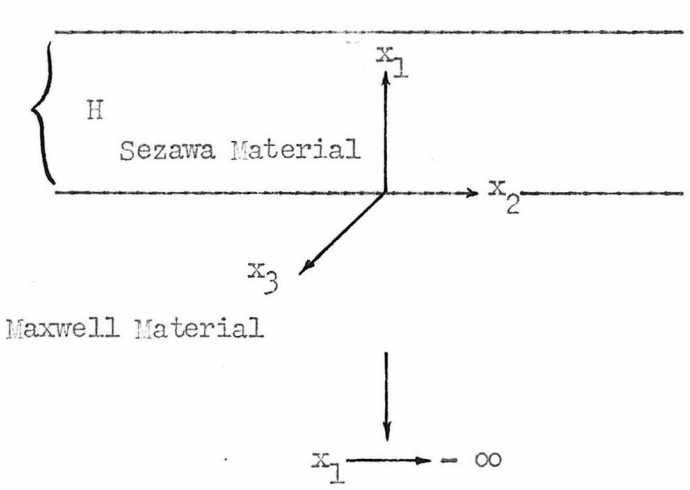

Figure 1

In course of the analysis the upper layer and lower layer will sometimes be referred to as the first and second layer, respectively.

By treating the material of the upper layer as perfectly elastic

it was found that the continuity conditions at the interface could not be satisfied. This can be attributed to wave attenuation produced in the direction of propagation (x_2) in the Maxwell material and to the absence of a mechanism (viscosity) for producing a corresponding attenuation in the perfectly elastic material. Since the assumption of perfect elasticity constitutes an idealization rather than a preferred representation of the physical properties of the shallow materials of the crust it was decided to represent them here by a Visco-Elastic (Sezawa) material.

Equations of Motion for the Visco-Elastic Layer: The equilibrium equations for the Visco-Elastic Layer are

$$\sum_{k=1}^{3} \frac{\partial S_{ik}}{\partial x_k} = \rho \frac{\partial^2 u_i}{\partial t^2} \qquad (i\&k = 1,2,3) \tag{1}$$

where:

S_{ik} are the components of the Stress Tensor

u_i are the components of the displacement Vector

We shall hence-forth neglect the $\sum$, with the understanding that when an index is repeated (such as k) summation over the index is implied. For an incompressible Sezawa material, $(\frac{\partial u_k}{\partial x_k} = 0)$ the stress-strain-time relations are given by reference (33)

$$S_{ik} = \mu \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right) + \sigma \frac{\partial}{\partial t}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right) \tag{2}$$

Then,

$$\frac{\partial S_{ik}}{\partial x_k} = \mu\left[\frac{\partial^2 u_i}{\partial x_k \partial x_k} + \frac{\partial}{\partial x_i}\left(\frac{\partial u_k}{\partial x_k}\right)\right] + \sigma \frac{\partial}{\partial t}\left[\frac{\partial^2 u_i}{\partial x_k \partial x_k} + \frac{\partial}{\partial x_i}\left(\frac{\partial u_k}{\partial x_k}\right)\right] \tag{2a}$$

Since $\frac{\partial u_k}{\partial x_k} = 0$ equation (2) becomes

$$\frac{\partial S_{ik}}{\partial x_k} = (\mu + \sigma \frac{\partial}{\partial t}) \frac{\partial^2 u_i}{\partial x_k \partial x_k} \tag{2b}$$

Substituting (2b) in (1) gives

$$\frac{\partial^2 u_i}{\partial t^2} = \frac{\mu}{\rho} \nabla^2 u_i + \frac{\sigma}{\rho} \frac{\partial}{\partial t} \nabla^2 u_i \quad (i = 1,2,3) \tag{3}$$

Consider a solution to (3) of the form

$$u_3 = \psi(x_1, x_2) T(t) e^{\bar{i}(fx_2 + pt + \varepsilon)} \tag{4}$$

and let

$$u_1 = u_2 = 0$$

Solution (4) represents a progressive wave in x_2, in which the functions ψ and T are treated as arbitrary.

From (4)

$$\left.\begin{aligned} \frac{\partial u_3}{\partial t} &= \psi e^{\bar{i}(fx_2 + pt + \varepsilon)} [\bar{i}pT + T'] \\ \frac{\partial^2 u_3}{\partial t^2} &= \psi e^{\bar{i}(fx_2 + pt + \varepsilon)} [T'' + 2\bar{i}pT' - p^2 T] \\ \frac{\partial^2 u_3}{\partial x_1^2} &= \frac{\partial^2 \psi}{\partial x_1^2} T e^{\bar{i}(fx_2 + pt + \varepsilon)} \end{aligned}\right\} \tag{5}$$

$$\left.\begin{aligned}
\frac{\partial^2 u_3}{\partial x_2^2} &= T e^{\bar{i}(fx_2+pt+\varepsilon)}\left[\frac{\partial^2\psi}{\partial x_2^2} + 2\bar{i}f\frac{\partial\psi}{\partial x_2} - f^2\psi\right] \\
\frac{\partial}{\partial t}\left(\frac{\partial^2 u_3}{\partial x_1^2}\right) &= \frac{\partial^2\psi}{\partial x_1^2} e^{\bar{i}(fx_2+pt+\varepsilon)}\left[\bar{i}pT + T'\right] \\
\frac{\partial}{\partial t}\left(\frac{\partial^2 u_3}{\partial x_2^2}\right) &= \left(\frac{\partial^2\psi}{\partial x_2^2} + 2\bar{i}f\frac{\partial\psi}{\partial x_2} - f^2\psi\right) e^{\bar{i}(fx_2+pt+\varepsilon)}\left[\bar{i}pT + T'\right]
\end{aligned}\right\} \quad (5)$$

Substituting (5) in (3) gives

$$\left\{\psi(T'' + 2\bar{i}pT' - p^2T) - \frac{\mu}{\rho}T\left(\frac{\partial^2\psi}{\partial x_1^2} + \frac{\partial^2\psi}{\partial x_2^2} + 2\bar{i}f\frac{\partial\psi}{\partial x_2} - f^2\psi\right)\right.$$

$$\left. - \frac{\sigma}{\rho}\left[(\bar{i}pT + T')\left(\frac{\partial^2\psi}{\partial x_1^2} + \frac{\partial^2\psi}{\partial x_2^2} + 2\bar{i}f\frac{\partial\psi}{\partial x_2} - f^2\psi\right)\right]\right\} e^{\bar{i}(fx_2+pt+\varepsilon)} = 0 \quad (6)$$

Satisfying the real and imaginary parts of equation (6) separately, gives two equations for the functions ψ and T.

$$\psi T'' - \psi p^2 T - \frac{\mu}{\rho}T\left(\frac{\partial^2\psi}{\partial x_1^2} + \frac{\partial^2\psi}{\partial x_2^2} - f^2\psi\right) - \frac{\sigma}{\rho}\left[T'\left(\frac{\partial^2\psi}{\partial x_1^2} + \frac{\partial^2\psi}{\partial x_2^2}\right.\right.$$

$$\left.\left. - f^2\psi\right) - 2pfT\frac{\partial\psi}{\partial x_2}\right] = 0 \quad (7)$$

$$2\psi pT' - 2\frac{\mu}{\rho}Tf\frac{\partial\psi}{\partial x_2} - \frac{\sigma}{\rho}\left[2fT'\frac{\partial\psi}{\partial x_2} + pT\left(\frac{\partial^2\psi}{\partial x_1^2} + \frac{\partial^2\psi}{\partial x_2^2} - f^2\psi\right)\right] = 0 \quad (8)$$

From (7) one obtains for $\nabla^2\psi = \frac{\partial^2\psi}{\partial x_1^2} + \frac{\partial^2\psi}{\partial x_2^2}$

$$\nabla^2\psi = \frac{\rho}{(\mu T+\sigma T')}\left[\psi T'' - \psi p^2 T + \frac{\mu}{\rho} f^2 T\psi + \frac{\sigma}{\rho} f^2 T'\psi + 2pf\frac{\sigma}{\rho} T\frac{\partial\psi}{\partial x_2}\right] \quad (7a)$$

From equation (8) one obtains

$$\nabla^2\psi = \frac{\rho}{\sigma pT}\left[2p\,T'\psi - 2\frac{\mu}{\rho} fT\frac{\partial\psi}{\partial x_2} - 2\frac{\sigma}{\rho} fT'\frac{\partial\psi}{\partial x_2} + \frac{\sigma}{\rho} f^2 pT\psi\right] \quad (8a)$$

Eliminating $\nabla^2\psi$ between (7a) and (8a) yields

$$\frac{\partial\psi}{\partial x_2}\left(\frac{2\frac{\mu}{\rho}f}{\sigma p} + \frac{2pf\frac{\sigma}{\rho}T}{\mu T+\sigma T'} + \frac{2fT'}{\rho pT}\right) = \left\{\frac{1}{\mu T+\sigma T'}\left[-T'' + p^2T - \frac{\mu}{\rho} f^2T - \frac{\sigma}{\rho} f^2T'\right] + \frac{1}{\sigma pT}\left[2pT' + \frac{\sigma}{\rho} f^2 pT\right]\right\}\psi \quad (9)$$

Symbolically (9) is of the form

$$\frac{\partial\psi}{\partial x_2} = \frac{\frac{1}{\mu T+\sigma T'}\left[-T'' + p^2T - \frac{\mu}{\rho} f^2T - \frac{\sigma}{\rho} f^2T'\right] + \frac{1}{\sigma pT}\left[2pT' + \frac{\sigma}{\rho} f^2 pT\right]}{\left(\frac{2\frac{\mu}{\rho}f}{\sigma p} + \frac{2pf\frac{\sigma}{\rho}T}{\mu T+\sigma T'} + \frac{2fT'}{\rho pT}\right)}\psi \quad (9a)$$

and therefore

$$\psi = \Phi(x_1)e^{Kx_2} \quad (10)$$

where

$$K = \frac{\frac{1}{\mu T+\sigma T'}\left[-T'' + p^2T - \frac{\mu}{\rho} f^2T - \frac{\sigma}{\rho} f^2T'\right] + \frac{1}{\sigma pT}\left[2pT' + \frac{\sigma}{\rho} f^2 pT\right]}{\left(\frac{2\frac{\mu}{\rho}f}{\sigma p} + \frac{2pf\frac{\sigma}{\rho}T}{\mu T+\sigma T'} + \frac{2fT'}{\rho pT}\right)}$$

and $\Phi(x_1)$ is an arbitrary function.

It is important to notice that in general $K = K(t)$, unless $T(t) \sim e^{mt}$ where m is a constant. Otherwise

$$\psi = \Phi(x_1)e^{K(t)x_2} = \psi(x_1 x_2 t)$$

which contradicts (4) according to which ψ has been treated mathematically as a function of x_1 and x_2 only. Thus we consider T(t) of the form

$$T(t) \sim e^{mt} \tag{11}$$

Then, in equation (9a) we have

$$\left(\frac{2\frac{\mu}{\rho}f}{\sigma p} + \frac{2pf\frac{\sigma}{\rho}T}{\mu T+\sigma T'} + \frac{2fT'}{\rho pT}\right) = \frac{2\mu f}{\sigma\rho p} + \frac{2pf\sigma}{\rho(\mu+m\sigma)} + \frac{2fm}{\rho p}$$

$$= \frac{2f}{\rho p}\left(\frac{\mu}{\sigma} + \frac{p^2\sigma}{(\mu+m\sigma)} + m\right) \tag{9a'}$$

and

$$\frac{1}{\mu T+\sigma T'}\left[-T'' + p^2T - \frac{\mu}{\rho}f^2T - \frac{\sigma}{\rho}f^2T'\right] + \frac{1}{\sigma pT}\left[2pT' + \frac{\sigma}{\rho}f^2pT\right] =$$

$$= \frac{1}{\mu+m\sigma}\left[-m^2 + p^2 - \frac{\mu}{\rho}f^2 - \frac{\sigma}{\rho}mf^2\right] + \frac{1}{\sigma p}\left(2pm + \frac{\sigma}{\rho}f^2p\right) \tag{9a''}$$

Combining equations (9a') and (9a'') gives for K in equation (10)

$$K = \frac{\frac{1}{\mu+m\sigma}\left(-m^2 + p^2 - \frac{\mu}{\rho}f^2 - \frac{\sigma}{\rho}mf^2\right) + \frac{2m}{\sigma} + \frac{f^2}{\rho}}{\frac{2f}{\rho p}\left[\frac{\mu}{\sigma} + \frac{p^2\sigma}{(\mu+m\sigma)} + m\right]} \tag{12}$$

Substituting (9a) into (7a) yields

$$\frac{d^2\Phi}{dx_1^2} + K^2\Phi = \frac{\rho}{\mu+\sigma m}\left(\Phi m^2 - \Phi p^2 + \frac{\mu}{\rho} f^2\Phi + \frac{\sigma}{\rho} f^2 m\Phi + 2pfK\frac{\sigma}{\rho}\Phi\right)$$

or

$$\frac{d^2\Phi}{dx_1^2} + B\Phi = 0 \tag{13}$$

where

$$B = K^2 - \frac{\rho}{\mu+m\sigma}\left(m^2 - p^2 + \frac{\mu}{\rho} f^2 + \frac{\sigma}{\rho} f^2 m + 2pf\frac{\sigma}{\rho}K\right)$$

The solution to (13) is of the form

$$\Phi(x_1) = C\cos\sqrt{B}x_1 + D\sin\sqrt{B}x_1 \tag{14}$$

where:

$$B = K^2 - \frac{\rho}{\mu+m\sigma}\left(m^2 - p^2 + \frac{\mu}{\rho} f^2 + \frac{\sigma}{\rho} f^2 m + 2pf\frac{\sigma}{\rho}K\right) > 0$$

C and D are arbitrary constants. The condition $B > 0$ is imposed here in order to satisfy the boundary condition of vanishing stress at the upper surface, which gives the trigonometric solution (14) to equation (13).

Using equations 10, 11, 14, and 4 one obtains the following solution for u_3.

$$u_3 = (C\cos\sqrt{B}x_1 + D\sin\sqrt{B}x_1)\, e^{Kx_2}\, e^{mt}\, e^{i(fx_2+pt+\varepsilon)} \tag{15}$$

where:

a) m is a constant subject to the condition $m < 0$ and remains

to be determined.

b) f, p and ε have their usual physical meaning in the mathematical representation of a single harmonic progressive wave.

c) The quantity K is defined by (12)

The Differential Equations of Motion for the Plastic Layer: The principle of conservation of momentum, neglecting body forces gives

$$\sum_{k=1}^{3} \frac{\partial S'_{ik}}{\partial x_k} = \rho' \frac{\partial^2 u'_i}{\partial t^2} \quad (i = 1,2,3) \tag{13}$$

where:

a) S'_{ik} are the components of the stress tensor in the plastic layer.

b) u'_i $(i = 1,2,3)$ are the components of the displacement vector.

c) ρ' is the density of the plastic material.

d) x_i $(i = 1,2,3)$ are rectangular cartesian coordinates.

The stress-strain-time relations for a Maxwell material are

$$\dot{\varepsilon}'_{ik} = \frac{S'_{ik}}{2\sigma'} + \frac{1}{2\mu'} \dot{S}'_{ik}$$

or

$$S'_{ik} = \dot{\varepsilon}'_{ik}\, 2\sigma' - \frac{\sigma'}{\mu'} S'_{ik} \tag{14}$$

where

$$\cdot \equiv \frac{\partial}{\partial t}$$

Substituting (14) into (13) gives for incompressible (constant volume) deformation where $\Delta = \dot{\Delta} = \sum_{i=1}^{3} \frac{\partial \dot{u}_i'}{\partial x_i} = \frac{\partial \dot{u}_i'}{\partial x_i} = 0$

$$\sigma' \nabla^2 \dot{u}_i' + b \sum_{k=1}^{3} \left(\frac{\partial \dot{S}_{ik}'}{\partial x_k}\right) = \rho' \frac{\partial \dot{u}_i'}{\partial t}, i = (1,2,3) \tag{15}$$

where:

$$b = -\frac{\sigma'}{\mu'}$$

$$\nabla^2 = \left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}\right)$$

Using equation (13) in (15) yields

$$\sigma' \nabla^2 \dot{u}_i' + b\rho' \frac{\partial^2 \dot{u}_i'}{\partial t^2} = \rho' \frac{\partial \dot{u}_i'}{\partial t} \quad (i = 1,2,3) \tag{16}$$

Although more general solutions are evidently possible, in this investigation we will assume $u_1' = u_2' = 0$ and that u_3' is of the form

$$u_3' = \psi(x_1, x_2)\, e^{\bar{i}(fx_2 + pt + \varepsilon)} \tag{17}$$

where:

$\psi(x_1, x_2)$ is presently an unknown function,

Substituting (17) in (16) yields

$$\left.\begin{aligned} \frac{\partial^2 u_3'}{\partial x_1^2} &= \frac{\partial^2 \psi}{\partial x_1^2} e^{\bar{i}(fx_2 + pt + \varepsilon)} \\ \frac{\partial u_3'}{\partial x_2} &= \frac{\partial \psi}{\partial x_2} e^{\bar{i}(fx_2 + pt + \varepsilon)} + \bar{i} f \psi e^{\bar{i}(fx_2 + pt + \varepsilon)} \end{aligned}\right\} \tag{18}$$

$$= e^{\bar{i}(fx_2+pt+\varepsilon)}\left[\frac{\partial\psi}{\partial x_2} + \bar{i}f\psi\right]$$

$$\frac{\partial^2 u_3'}{\partial x_2^2} = e^{\bar{i}(fx_2+pt+\varepsilon)}\left[\frac{\partial^2\psi}{\partial x_2^2} + 2if\frac{\partial\psi}{\partial x_2} - f^2\psi\right] \qquad (18)$$

$$\frac{\partial^2 u_3'}{\partial x_3^2} = 0$$

and

$$\frac{\partial^3 u_3'}{\partial t^3} = -\bar{i}p^3\psi e^{\bar{i}(fx_2+pt+\varepsilon)}$$

$$\frac{\partial^2 u_3'}{\partial t^2} = -p^2\psi e^{\bar{i}(fx_2+pt+\varepsilon)} \qquad (19)$$

From (18)

$$\frac{\partial^2 \dot{u}_3'}{\partial x_1^2} = \bar{i}p\frac{\partial^2\psi}{\partial x_1^2}e^{\bar{i}(fx_2+pt+\varepsilon)}$$

$$\frac{\partial^2 \dot{u}_3'}{\partial x_2^2} = \bar{i}p\left[\frac{\partial^2\psi}{\partial x_2^2} + 2\bar{i}f\frac{\partial\psi}{\partial x_2} - f^2\psi\right]e^{\bar{i}(fx_2+pt+\varepsilon)}$$

$$= \left[\bar{i}p\frac{\partial^2\psi}{\partial x_2^2} - 2fp\frac{\partial\psi}{\partial x_2} - \bar{i}f^2p\psi\right]e^{i(fx_2+pt+\varepsilon)} \qquad (18a)$$

Substituting (18a) and (19) into (16) yields for i = 3

$$\sigma'\left\{ip\frac{\partial^2\psi}{\partial x_1^2} + ip\frac{\partial^2\psi}{\partial x_2^2} - 2fp\frac{\partial\psi}{\partial x_2} - \bar{i}f^2\psi p\right\} - b\rho'\bar{i}p^3\psi + \rho'(+p^2\psi) \equiv 0 \qquad (20)$$

Let $p = p_1 + \bar{i}p_2$ where p_1 and p_2 are real numbers.

Then $p^2 = p_1^2 + 2ip_1p_2 - p_2^2$

$$p^3 = p_1^3 + 3\bar{i}p_1^2p_2 - 3p_1p_2^2 - ip_2^3 \tag{21}$$

Substituting (21) into (20) gives

$$\sigma'\left[\frac{\partial^2\psi}{\partial x_1^2}(\bar{i}p_1 - p_2) + \frac{\partial^2\psi}{\partial x_2^2}(\bar{i}p_1 - p_2) - \frac{\partial\psi}{\partial x_2}(2fp_1 + 2f\bar{i}p_2)\right.$$

$$\left. - f^2\psi(\bar{i}p_1 - p_2)\right] - b\rho'\psi(\bar{i}p_1^3 - 3p_1^2p_2 - 3\bar{i}p_1p_2^2 + p_2^3)$$

$$+\rho'\psi(p_1^2 + 2\bar{i}p_1p_2 - p_2^2) \equiv 0 \tag{22}$$

The real and pure imaginary parts of (22) give respectively

$$-\frac{\partial^2\psi}{\partial x_1^2} - \frac{\partial^2\psi}{\partial x_2^2} - 2f\frac{p_1}{p_2}\frac{\partial\psi}{\partial x_2} + f^2\psi + 3p_1^2\frac{b\rho'}{\sigma'}\psi - p_2^2\frac{b\rho'}{\sigma'}\psi$$

$$+\frac{p_1^2}{p_2}\frac{\rho'\psi}{\sigma'} - \frac{\rho'p_2}{\sigma'}\psi = 0 \tag{23}$$

$$\frac{\partial^2\psi}{\partial x_1^2} + \frac{\partial^2\psi}{\partial x_2^2} - 2f\frac{p_2}{p_1}\frac{\partial\psi}{\partial x_2} - f^2\psi - p_1^2\frac{b\rho'}{\sigma'}\psi + 3p_2^2\frac{b\rho'}{\sigma'}\psi$$

$$+ 2p_2\frac{\rho'}{\sigma'}\psi = 0 \tag{24}$$

Equations (23) and (24) can be treated as a set of simultaneous differential equations for the function

$$\psi(x_1 x_2)$$

Using $\frac{b}{\sigma'} = \frac{1}{\mu'}$ these become respectively

$$\nabla^2\psi = -2f\frac{p_1}{p_2}\frac{\partial\psi}{\partial x_2} + \psi[f^2 - 3p_1^2\frac{\rho'}{\mu'} + p_2^2\frac{\rho'}{\mu'} + \frac{p_1^2}{p_2}\frac{\rho'}{\sigma'} - p_2\frac{\rho'}{\sigma'}] \tag{23a}$$

$$\nabla^2\psi = 2f\frac{p_2}{p_1}\frac{\partial\psi}{\partial x_2} + \psi[f^2 - p_1^2\frac{\rho'}{\mu'} + 3p_2^2\frac{\rho'}{\mu'} - 2p_2\frac{\rho'}{\sigma'}] \tag{24a}$$

Eliminating $\nabla^2\psi$ between (23a) and (24a) gives

$$\frac{\partial\psi}{\partial x_2} = [-\frac{\rho'}{f\mu'}p_1p_2 + \frac{\rho'(p_1^3+p_1p_2^2)}{2\sigma' f(p_1^2+p_2^2)}]\psi \tag{25}$$

A solution to (25) is

$$\psi = \Phi(x_1)e^{Kx_2} \tag{26}$$

where

$$K = -\frac{\rho'}{f\mu'}p_1p_2 + \frac{\rho'(p_1^3+p_1p_2^2)}{2f\sigma'(p_1^2+p_2^2)}$$

and $\Phi(x_1)$ is an arbitrary function of x_1.

Substituting (26) into either (23) or (24) yields a linear, second order, homogeneous, total differential equation for the function $\Phi(x_1)$. A substitution of (26) in (23) gives the differential equation

$$\frac{d^2\Phi}{dx_1^2} - c_1^2\Phi = 0 \tag{27}$$

where:

$$c_1^2 = \left[- K^2 - 2Kf \frac{p_1}{p_2} + f^2 - 3p_1^2 \frac{\rho'}{\mu'} + p_2^2 \frac{\rho'}{\mu'} + \frac{p_1^2}{p_2} \frac{\rho'}{\sigma'} - \frac{p_2 \rho'}{\sigma'}\right]$$

The solution to (27) is therefore of the form

$$\Phi(x_1) = A' e^{c_1 x_1} + B' e^{-c_1 x_1} \tag{28}$$

where:

c_1 is defined above, and A' and B' are arbitrary constants.

A substitution of (26) into (24) gives

$$\frac{d^2\Phi}{dx_1^2} - c_2^2 \Phi = 0 \tag{29}$$

where:

$$c_2^2 = \left[- K^2 + 2Kf \frac{p_2}{p_1} + f^2 - p_1^2 \frac{\rho'}{\mu'} + 3p_2^2 \frac{\rho'}{\mu'} - 2p_2 \frac{\rho'}{\sigma'}\right]$$

in order that (27) and (28) be identical it is necessary that

$$c_1^2 = c_2^2 \tag{30}$$

Equation (30) requires that K be

$$K = - \frac{\rho'}{\mu' f} p_1 p_2 + \frac{\rho'(p_1^3 + p_1 p_2^2)}{2\sigma' f(p_1^2 + p_2^2)} \tag{31}$$

The expression for K given by (31) is identical with that given by (26) which means that the condition $c_1^2 = c_2^2$ is satisfied and therefore that the solution for Φ can also be written as

$$\Phi(x_1) = A' e^{c_2 x_1} + B' e^{-c_2 x_1} \tag{32}$$

where:

c_2^2 is given above, and A' and B' are arbitrary constants.

Using (32), (26) and (17), a solution to (16) can be written in the form

$$u_i' = A_i e^{c_{ij} x_j} e^{-r_i t} e^{\bar{i}(fx_2 + pt + \varepsilon)} \qquad (i,j = 1,2,3) \tag{33}$$

where:

A_i is an arbitrary constant.

r_i is a constant to be determined and is here subjected to the condition $r_i \geq 0$.

$$c_{i1} = \sqrt{- c_{i2}^2 + 2c_{i2} f \frac{r_i}{p} + f^2 - p^2 \frac{\rho'}{\mu'} + 3r_i^2 \frac{\rho'}{\mu'} - 2r_i \frac{\rho'}{\sigma'}}$$

and $c_{i1}^2 > 0$.

$$c_{i2} = \left(- \frac{\rho'}{\mu' f} r_i p + \frac{\rho' p}{2\sigma' f}\right) < 0.$$

$c_{i3} = 0$. (Hence $c_{ij} \neq c_{ji}$)

$\bar{i} = \sqrt{-1}$

In order to prescribe the boundary conditions at the interface

between the visco-elastic and plastic layers, one must relate the stress tensor S'_{ik} to the displacement vector u'_i. This can be done by first integrating the following equation, relating the strain and stress tensors of a Maxwell material

$$\frac{\partial}{\partial t}(\varepsilon'_{ik}) = \frac{1}{2\sigma'} S'_{ik} + \frac{1}{2\mu'} \frac{\partial}{\partial t}(S'_{ik}) \tag{34}$$

where: ε'_{ik} is the strain tensor of the Maxwell material given by

$$\varepsilon'_{ik} = \frac{1}{2}\left(\frac{\partial u'_i}{\partial x_k} + \frac{\partial u'_k}{\partial x_i}\right) \qquad (i,k = 1,2,3) \tag{35}$$

Substituting (33) into (35) gives

$$\varepsilon'_{ik} = \frac{1}{2}(c_{ik}u'_i + c_{ki}u'_k)$$

$$\frac{\partial}{\partial t}(\varepsilon'_{ik}) = \frac{1}{2}[c_{ik}(-r_i + \bar{i}p)u'_i + c_{ki}(-r_k + \bar{i}p)u'_k]$$

or

$$\frac{\partial}{\partial t}(\varepsilon'_{ik}) = \frac{1}{2}\left\{[c_{ik}(-r_i + \bar{i}p)A_i C^{c_{ij}x_j - r_i t} + c_{ki}(-r_k + \bar{i}p) \times \right.$$

$$\left. \times A_k e^{c_{kj}x_j - r_k t}]e^{\bar{i}(fx_2 + pt + \varepsilon)}\right\} \tag{36}$$

Substituting (34) into (36) and arranging terms gives

$$\frac{\partial}{\partial t}(S'_{ik}) + \frac{\mu'}{\sigma'} S'_{ik} = \mu'\left\{[c_{ik}(-r_i + \bar{i}p)A_i e^{c_{ij}x_j - r_i t}\right.$$

$$+ c_{ki}(- r_k + \bar{i}p)A_k e^{c_{kj}x_j - r_k t}]e^{\bar{i}(fx_2+pt+\varepsilon)}\Bigg\} \qquad (37)$$

For each i and k equation (37) is a linear, first order differential equation of the form

$$\frac{dy}{dx} + P(x)y = Q(x) \qquad (38)$$

where

$$P(x) \equiv P(t) = \frac{\mu'}{\sigma'}$$

$$Q(x) \equiv Q(t) = \mu'\{\quad\}$$

The solution to (38) is of the form

$$ye^{\int pdx} = \int Qe^{\int pdx}dx + K \qquad (39)$$

where K is an arbitrary constant.

Hence, the solution to (37) is

$$S'_{ik}e^{\frac{\mu'}{\sigma'}t} = \mu'\int\{\quad\}e^{\frac{\mu'}{\sigma'}t}\,dt + K_{ik}(x_1,x_2)$$

where $K_{ik}(x_1,x_2)$ is an arbitrary function or

$$S'_{ik} = \mu' e^{-\frac{\mu'}{\sigma'}t}\int\{\quad\}e^{\frac{\mu'}{\sigma'}t}\,dt + K_{ik}e^{-\frac{\mu'}{\sigma'}t} \qquad (40)$$

carrying out the integration in (40) gives

$$S'_{ik} = \mu'\sigma'\Bigg\{ e^{c_{ij}x_j}e^{\bar{i}(fx_2+pt+\varepsilon)}\left[\frac{c_{ik}(-r_i+\bar{i}p)A_i e^{-r_i t}}{\sigma'(-r_i+\bar{i}p)+\mu'}\right]$$

$$+ e^{c_{kj}x_j} e^{\bar{i}(fx_2+pt+\varepsilon)} \left[\frac{c_{ki}(-r_k+\bar{i}p)A_k e^{-r_k t}}{\sigma'(-r_k+\bar{i}p)+\mu'}\right] \Bigg\}$$

$$+ K_{ik}(x_1,x_2)e^{-\frac{\mu'}{\sigma'}t} \tag{41}$$

The results thus far obtained in the analysis are sufficiently general for treating analytically the propagation of a three component wave in direction x_3.

Case I.

On the Propagation of Space-Attenuated, Time Unattenuated Waves:

We will now restrict the analysis to the propagation of waves with a displacement pattern given by $u_1' = u_2' = 0$ and where $u_3' \neq 0$ is obtained from equation (33) by setting $i = 3$. The waves thus conditioned are therefore, at the surface, typical of Love Waves. It is now significant to determine whether or not a steady, simple harmonic progressive wave, attenuating spatially in the direction of propagation, can be propagated as a surface wave through the configuration and media described by figure 1. Thus in equation (33) we set $r_3 = 0$ and obtain with $u_1' = u_2' = 0$ the relation

$$u_3' = A_3 e^{c_{31}x_1} e^{c_{32}x_2} e^{\bar{i}(fx_2+pt+\varepsilon)} \tag{42}$$

where:

A_3 is an arbitrary constant.

$$c_{31} = \sqrt{-c_{32}^2 + f^2 - p^2\frac{\rho'}{\mu'}} \quad \text{and} \quad c_{31}^2 > 0$$

$$c_{32} = \frac{\rho' p}{2\sigma' f} < 0$$

Boundary and Other Conditions: The boundary conditions are:

a) Continuity of displacements at the interface ($x_1 = 0$), separating the visco-elastic and plastic materials. This requires that $u_3 = u_3'$.

b) That the upper surface ($x_1 = H$) of the visco-elastic material be unstressed (vacuum). This leads to the condition $S_{13} = 0$ at $x_1 = H$, which can be modified, if desirable, to account for the surface stresses produced by the atmosphere.

c) Continuity of stresses at the interface ($x_1 = 0$). This requires that $S_{13} = S_{13}'$ at $x_1 = 0$.

d) Since we are here considering surface waves it is further required that u_3' attenuate with depth, i.e., $c_{31} > 0$.

e) It is also required here that $K \leqq 0$.

Using equations (15), (42) and boundary condition (a), one obtains

$$A_3 e^{c_{32}x_2} e^{i(fx_2+pt+\varepsilon)} = Ce^{Kx_2} e^{mt} e^{i(fx_2+pt+\varepsilon)} \qquad (43)$$

In order that (43) be identically satisfied it is necessary that

$$\begin{aligned} A_3 &= C \\ c_{32} &= K \\ m &= 0 \end{aligned} \qquad (43a)$$

Using boundary condition (b) in conjunction with equations (2) and (15) yields the relation

$$\frac{D}{C} = \tan \sqrt{B}H \tag{44}$$

Imposing boundary condition (c) and using equation (41) with i = 1 and k = 3, and noting that $A_i = A_1 = 0$ (since $u_1 = 0$) and $r_i = r_k = 0$ yields

$$S'_{13} = \mu'\sigma' e^{(c_{31}x_1 + c_{32}x_2)} e^{\bar{i}(fx_2+pt+\varepsilon)} \cdot \frac{c_{31}(\bar{i}p)A_3}{\sigma'\bar{i}p+\mu'}$$

$$+ K_{13}(x_1,x_2)e^{-\frac{\mu'}{\sigma'}t} \tag{45}$$

For the visco-elastic material we have the relation (equation 2)

$$S_{13} = \mu \frac{\partial u_3}{\partial x_1} + \sigma \frac{\partial}{\partial t}\left(\frac{\partial u_3}{\partial x_1}\right) \tag{46}$$

Substituting (15) in (46) and using m = 0, (see equation 3a) yields

$$S_{13} = e^{Kx_2} e^{mt} e^{\bar{i}(fx_2+pt+\varepsilon)} \sqrt{B}\,[-C \sin \sqrt{B}x_1 + D \cos \sqrt{B}x_1]\left\{\mu + \sigma(\bar{i}p)\right\} \tag{47}$$

Equating (45) and (47) at $x_1 = 0$ and using $c_{32} = K$ one obtains

$$e^{c_{32}x_2} e^{\bar{i}(fx_2+pt+\varepsilon)} \left\{ \frac{\bar{i}pc_{31}A_3}{\bar{i}\sigma'p+\mu'} \cdot \mu'\sigma' - \sqrt{B}D\,[\mu + \bar{i}\sigma p] \right\}$$

$$+ K_{13}(0,x_2)e^{-\frac{\mu'}{\sigma'}t} = 0 \tag{48}$$

In order that (48) be satisfied at all times it is necessary that $K_{13}^{*}(0,x_2)$ and the term $\left\{\quad\right\}$ vanish separately. Thus equating the term $\left\{\quad\right\}$ to zero yields

$$\frac{\mu'\sigma' p c_{31} A_3 (\sigma' p + \bar{i}\mu')}{\mu'^2 + p^2\sigma'^2} = \sqrt{BD}\,[\mu + \bar{i}\sigma p] \tag{49}$$

Equating the real and pure imaginary expressions in (49) gives

$$\frac{\mu' p^2 c_{31} A_3 \sigma'^2}{p^2\sigma'^2 + \mu'^2} = \sqrt{BD}\,\mu \tag{50}$$

$$\frac{\sigma' p c_{31} A_3 \mu'^2}{p^2\sigma'^2 + \mu'^2} = \sqrt{BD}\,\sigma p \tag{51}$$

One obtains from (50) and (51) by dividing the relations

$$\frac{p\sigma'}{\mu'} = \frac{\mu}{\sigma}\frac{1}{p}$$

or

$$p = \sqrt{\frac{\mu\mu'}{\sigma'\sigma}} \tag{52}$$

Equation (52) relates the admissible frequencies p to the physical constants of the plastic and visco-elastic materials. Imposing now the condition $c_{32} = K$, where c_{32} and K are respectively given by (42) and (12), yield with m = 0

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* This does not require however that $K_{13}(x_1 x_2) \equiv 0$ for $x_1 \neq 0$. Therefore the boundary condition at the interface permits the superposition of an arbitrary stress distribution in the plastic sub-layer. From (45) it is seen however that this stress appears as a coefficient of a transient term in (45) and can therefore be neglected in the investigation of steady state phenomena.

$$\frac{\rho' p}{2\sigma' f} = \frac{\frac{1}{\mu}(p^2 - \frac{\mu}{\rho} f^2) + \frac{f^2}{\rho}}{\frac{2f}{\rho p}(\frac{\mu}{\sigma} + p^2 \frac{\sigma}{\mu})}$$

or

$$p = \frac{\mu}{\sigma} \frac{1}{\sqrt{(\frac{\rho}{\rho'})(\frac{\sigma'}{\sigma}) - 1}} \tag{53}$$

and for real p we then have the condition

$$(\frac{\rho}{\rho'})(\frac{\sigma'}{\sigma}) > 1 \tag{54}$$

Eliminating p between (52) and (53), one obtains the following equation, relating constants of the plastic and visco-elastic materials.

$$\frac{\mu'}{\mu} = \frac{\sigma'}{\sigma} \frac{1}{[(\frac{\rho}{\rho'})\frac{\sigma'}{\sigma} - 1]} \tag{55}$$

Thus we find that the material constants must satisfy equation (55) in order that wave forms defined by (15) and (42) can be propagated simultaneously in the visco-elastic and plastic layers respectively. Condition (e) and equation (43a) require that $K = c_{32} \leqq 0$. From equation (12) one thus obtains for $m = 0$

$$K = \frac{\rho p^3}{2f\mu} \frac{1}{(\frac{\mu}{\sigma} + p^2 \frac{\sigma}{\mu})} \leqq 0 \tag{56}$$

This condition is naturally satisfied since $f < 0$ and the other terms appearing on the right of (56) are all positive. Furthermore, if we impose the condition $B > 0$ we obtain

$$B = [K^2 - \frac{\rho}{\mu}(-p^2 + \frac{\mu}{\rho} f^2 + 2pf \frac{\sigma}{\rho} K)] > 0$$

or

$$[f^2 + 2pf\frac{\sigma}{\mu}K - (K^2 + \frac{\rho}{\mu}p^2)] < 0 \tag{57}$$

Now using equation (44), i.e., $\frac{D}{C} = \tan\sqrt{B}H$ and the real and imaginary parts of equation (49) yields

$$\frac{\mu' p^2 c_{31}\sigma'^2}{(\mu'^2 + p^2\sigma'^2)\sqrt{B}\mu} = \frac{D}{A_3} = \frac{D}{C} \tag{58}$$

$$\frac{\sigma' c_{31}\mu'^2}{(\mu'^2 + p^2\sigma'^2)\sqrt{B}\sigma} = \frac{D}{A_3} = \frac{D}{C}$$

Since $A_3 = C$

Combining equations (47) and (58) gives

$$\frac{p^2 c_{31}\sigma'^2\mu'}{(\mu'^2 + p^2\sigma'^2)\sqrt{B}\mu} = \tan\sqrt{B}H \tag{59}$$

$$\frac{c_{31}\mu'^2\sigma'}{(\mu'^2 + p^2\sigma'^2)\sqrt{B}\sigma} = \tan\sqrt{B}H$$

Expressions (50) are equivalent transcendental equations for the wave length $\frac{2\pi}{f}$, subject to the condition (57).

Summary:

For convenience the results of the analysis completed thus for Case I are summarized in the following. The admissible frequencies p (or periods) are determined by the relations

$$\text{(I)} \quad p = \sqrt{\frac{\mu\mu'}{\sigma'\sigma}}$$

$$\text{(II)} \quad p = \frac{\mu}{\sigma} \frac{1}{\sqrt{\frac{\rho}{\rho'} \cdot \frac{\sigma'}{\sigma} - 1}}$$

These give the following relation between the material constants

$$\text{(III)} \quad \frac{\mu'}{\mu} = \frac{\sigma'}{\sigma} \frac{1}{\left[\frac{\rho}{\rho'} \cdot \frac{\sigma'}{\sigma} - 1\right]}$$

For the wave length f we obtained the following equivalent transcendental equations

$$\text{(IV)} \quad \frac{p^2 c_{31} \sigma'^2 \sigma}{(\mu'^2 + p^2\sigma'^2)\sqrt{B}\mu} = \tan \sqrt{B}H \qquad \text{(a)}$$

$$\frac{c_{31}\mu'^2\sigma'}{(\mu'^2 + p^2\sigma'^2)\sqrt{B}\sigma} = \tan \sqrt{B}H \qquad \text{(b)}$$

(Either IV(a) or IV(b) can be used to calculate f)

where

$$\text{(a)} \quad c_{31} = \sqrt{-c_{32}^2 + f^2 - p^2\frac{\rho'}{\mu'}} \quad \text{and } c_{31}^2 > 0, \text{ this implies (b)}$$

(b) $$f^4 - p^2 \frac{\rho'}{\mu'} f^2 - \frac{\rho'^2 p^2}{4\sigma'^2} > 0$$

(c) $$c_{32} = \frac{\rho' p}{2\sigma' f}$$

(d) $$B = K^2 + \frac{\rho}{\mu} p^2 - f^2 - 2pf \frac{\sigma}{\mu} K > 0, \text{ this implies (e)}$$

(e) $$f^2 + 2pK \frac{\sigma}{\mu} f - (K^2 + \frac{\rho}{\mu} p^2) < 0$$

(f) $$K = \frac{\rho p^3}{2\mu f} \frac{1}{(\frac{\mu}{\sigma'} + p^2 \frac{\sigma}{\mu})}$$

It is proven in appendix 1 that the conditions imposed in the above analysis lead to the relation $c_{31}^2 = - B$. This relation is a contradiction since we had earlier imposed the conditions $B > 0$ and $c_{31} > 0$. This contradiction implies that the boundary conditions and other conditions imposed in this analysis do not permit the propagation in this configuration (figure 1) of a steady simple harmonic disturbance, in the form of a spacially attenuated, steady and undistorted simple harmonic progressive wave. This result does not necessarily detract from the geophysical significance of the physical configuration considered in the above analysis, since it suggests that a harmonic disturbance is distorted if propagated in such a configuration. The practical difficulties in identifying or tracing at different stations corresponding phases of a progressive surface wave are usually attributed to changes in amplitude and period during the propagation. Therefore physical configurations which naturally cause wave distortion may have more geological significance than those which do not distort simple harmonic propagation of disturbances.

Whether or not this is a valid interpretation of the physical significance of the above contradiction remains to be determined. It is planned to further investigate this question by obtaining more general solutions to the differential equations encountered here. A preliminary study of methods for extending the generality of the theory in this way has been made and shows that the Laplace Transform can be used for this purpose.

The trigonometric functions in x_1, appearing in expression (15) for u_3, were obtained by assuming apriori that the coefficient B, appearing in equation (13) is positive. By so doing the boundary conditions at the surface of the upper layer can be readily satisfied, and this yields a relation for calculating the wave length. In lieu of the relation $c_{31}^2 = -B$ cited above, which is a contradiction when $B > 0$, it seems advisable to investigate what happens when $B < 0$. In doing this the time as well as space attenuation exponents are retained for the purpose of generality. Before carrying out this investigation however, it is convenient to first consider a time attenuated surface wave based upon the expression obtained for u_3 when $B > 0$, which does not lead to mathematical contradiction.

Case II

The Propagation of a Time Attenuated Spacially Unattenuated Surface Wave: The expression for u_3' (plastic layer) is given by equation (33) where now r_3 is different from zero. Thus u_3' is given by

$$u_3' = A_3 e^{c_{31}x_1} e^{c_{32}x_2} e^{-r_3 t} e^{\bar{i}(fx_2+pt+\varepsilon)} \qquad (60)$$

where:

A_3 is an arbitrary constant

$$c_{31} = \sqrt{- c_{32}^2 + 2c_{32}f \frac{r_3}{p} + f^2 - p^2 \frac{\rho'}{\mu'} + 3r_3^2 \frac{\rho'}{\mu'} - 2r_3 \frac{\rho'}{\mu'}}$$

$$c_{31}^2 > 0$$

$$c_{32} = - \frac{\rho' r_3 p}{\mu' f} + \frac{\rho' p}{2\sigma' f} < 0$$

$$r_3 > 0$$

In the visco-elastic layer, u_3 is given by equation (15)

Enforcing the same boundary conditions used in the preceding analysis gives the following relations

a) The condition $u_3' = u_3$ at $x_1 = 0$ yields

$$A_3 e^{c_{32}x_2} e^{-r_3 t} e^{\bar{i}(fx_2+pt+\varepsilon)} = Ce^{Kx_2} e^{mt} e^{\bar{i}(fx_2+pt+\varepsilon)} \tag{61}$$

to satisfy (6) identically it is necessary

$$\begin{aligned} A_3 &= C \\ c_{32} &= K \\ -r_3 &= m \end{aligned} \tag{61a}$$

b) The condition $S_{13} = 0$ at $x_1 = H$ yields

$$\frac{D}{C} = \tan \sqrt{B}H \tag{62}$$

The shearing stress in the plastic layer is given by equation (41) with $i = 1$, $k = 3$, $A_i = A_1 = 0$.

This gives

$$S'_{13} = \mu'\sigma' e^{c_{31}x_1} e^{c_{32}x_2} e^{\bar{i}(fx_2+pt+\varepsilon)} \frac{c_{31}(-r_3+\bar{i}p)A_3 e^{-r_3 t}}{\sigma'(-r_3+ip)+\mu'} + K_{13}(x_1,x_2)e^{-\frac{\mu'}{\sigma'}t} \qquad (63)$$

The function $K_{13}(x_1,x_2)e^{-\frac{\mu'}{\sigma'}t}$ decreases exponentially with time and represents a relaxation of stress in the plastic layer. It is recalled that the mechanism for seismic activity proposed in Part IV of the thesis called for a stress relaxing mechanism in the plastic layer. The function $K_{13}(x_1,x_2)e^{-\frac{\mu'}{\sigma'}t}$ appearing in S'_{13} is an aspect of this mechanism. The present analysis refers to the particular time during the cycle of the seismic mechanism at which the residual stresses in the sub-layer are sufficiently relaxed and therefore sufficiently increased in the upper layer so to cause its acceleration (seismic disturbance). Therefore letting $K_{13}(0,x_2) = 0$ as required by equation (65), is consistent with the mechanism described in Part IV of the thesis.

The shearing stress in the visco-elastic layer is given by equation and is rewritten here for convenience

$$S_{13} = e^{Kx_2} e^{mt} e^{\bar{i}(fx_2+pt+\varepsilon)} \sqrt{B}\,[-C \sin \sqrt{B}x_1 + D \cos \sqrt{B}x_1][\mu + \sigma(m + \bar{i}p)] \qquad (64)$$

Equating (4) and (5) at the interface ($x_1 = 0$) gives

$$\mu'\sigma' e^{c_{32}x_2} e^{\bar{i}(fx_2+pt+\varepsilon)} e^{-r_3 t} \frac{c_{31}(-r_3+\bar{i}p)A_3}{\sigma'(-r_3+\bar{i}p)+\mu'} + K_{13}(0,x_2)e^{-\frac{\mu'}{\sigma'}t}$$

$$= e^{Kx_2} e^{mt} e^{\bar{i}(fx_2+pt+\varepsilon)} D\sqrt{B}\,[\mu + \sigma(m + \bar{i}p)] \tag{65}$$

It follows from equation (65) that $K_{13}(0,x_2) = 0$.

Substituting the following identity (66) into equation (65) and equating real and imaginary parts yields equations (67a) and (67b).

$$\frac{c_{31}(-r_3+\bar{i}p)A_3}{\sigma'(-r_3+\bar{i}p)+\mu'} \equiv \frac{c_{31}A_3(-r_3+\bar{i}p)}{(-\sigma'r_3+\mu')+ip\sigma'} \cdot \frac{(-\sigma'r_3+\mu')-\bar{i}p\sigma'}{(-\sigma'r_3+\mu')-\bar{i}p\sigma'}$$

$$= \frac{-c_{31}A_3r_3(-\sigma'r_3+\mu')+c_{31}A_3p^2\sigma'}{(-\sigma'r_3+\mu')^2+p^2\sigma'^2} + \frac{\bar{i}[c_{31}A_3p(-\sigma'r_3+\mu')+c_{31}A_3r_3p\sigma']}{(-\sigma'r_3+\mu')^2+p^2\sigma'^2} \tag{66}$$

$$\frac{\mu'\sigma'c_{31}A_3}{(-\sigma'r_3+\mu')^2+p^2\sigma'^2} \left\{ -r_3(-\sigma'r_3+\mu')+p^2\sigma' \right\} = \sqrt{B}D\,[\mu+m\sigma] \tag{67a}$$

$$\frac{\mu'\sigma'c_{31}A_3}{(-\sigma'r_3+\mu')^2+p^2\sigma'^2} \left\{ p\mu' \right\} = \sqrt{B}D\,[\sigma p] \tag{67b}$$

Dividing (67a) by (67b) and using $r_3 = -m$ gives

$$m(\sigma'm+\mu') + p^2\sigma' = \frac{\mu'}{\sigma}(\mu + m\sigma) \tag{68}$$

From equation (68) one obtains the relation

$$p^2 = \frac{\mu'}{\sigma}\left(\frac{\mu}{\sigma'} + m\frac{\sigma}{\sigma'}\right) - m\left(m + \frac{\mu'}{\sigma'}\right)$$

or

$$p = \sqrt{\frac{\mu'}{\sigma}\left(\frac{\mu}{\sigma'} + m\frac{\sigma}{\sigma'}\right) - m\left(m + \frac{\mu'}{\sigma'}\right)} \tag{69}$$

Equation (69) gives the frequency p as a function of the physical constants and damping exponent $m = -r_3$.

Using now the condition $c_{32} = K$ in accordance with (61a) and using the expressions for c_{32} and K, respectively given by equation (60) and (12), one ultimately obtains using $r_3 = -m$ the relation,

$$\frac{1}{\mu'\sigma'}\rho'(2m\sigma' + \mu')(\mu^2 + 2m\mu\sigma + p^2\sigma^2 + m^2\sigma^2)$$

$$= \rho\sigma(-m^2 + p^2) + 2m\rho(\mu + m\sigma) \tag{70}$$

Substituting in (70) the expression for p given by (69) ultimately yields the relation

$$m = -\frac{1}{2}\left(\frac{\mu}{2\sigma} + \frac{\mu'}{\sigma'} - \frac{1}{2}\frac{\rho}{\rho'}\frac{\mu'}{\sigma'}\right) \pm \frac{1}{2}\sqrt{\left(\frac{\mu}{2\sigma} + \frac{\mu'}{\sigma'} - \frac{1}{2}\frac{\rho}{\rho'}\frac{\mu'}{\sigma'}\right)^2 - \left(\frac{\mu'^2}{\sigma'^2} - \frac{\rho}{\rho'}\frac{\mu'^2}{\sigma'\sigma} + \frac{\mu\mu'}{\sigma'\sigma}\right)^2} \tag{71}$$

Equation (71) gives two values for m, both of which must be real, and at least one negative, in order that there be attenuation with time.

From equation (42) it follows that the condition that c_{31} be real requires

$$-c_{32}^2 + 2c_{32}f\frac{r_3}{p} + f^2 - p^2\frac{\rho'}{\mu'} + 3r_3^2\frac{\rho'}{\mu'} - 2r_3\frac{\rho'}{\sigma'} > 0 \tag{72}$$

We have from (60) the condition that $c_{32} \gtrless 0$. This gives

$$c_{32} = -\frac{\rho'}{\mu'_f} r_3 p + \frac{\rho' p}{2\sigma'_f} \gtrless 0$$

Using $m = -r_3$ this becomes

$$\frac{m}{\mu'_f} + \frac{1}{2\sigma'_f} \gtrless 0 \tag{73}$$

If we consider progressive waves propagating in the positive x_2 direction then $f < 0$, and we can write $f = -f'$ where $f' > 0$. Making this substitution in (73) gives with $f' > 0$ and $m < 0$ the inequality

$$0 > m \gtrless -\frac{\mu'}{2\sigma'} \tag{74}$$

Combining equations (67a), (67b) with equations (61a) and (62) gives the important relations

$$\frac{\mu'\sigma' c_{31}}{(m\sigma'+\mu')^2+p^2\sigma'^2} [m(m\sigma'+\mu') + p^2\sigma'] \frac{1}{\sqrt{B}(\mu+m\sigma)}$$

$$= \frac{D}{A_3} = \frac{D}{C} = \tan \sqrt{BH} \tag{76a}$$

$$\frac{\mu'\sigma' c_{31}}{(m\sigma'+\mu')^2+p^2\sigma'^2} \cdot \frac{\mu'}{\sqrt{B}\sigma} = \frac{D}{A_3} = \frac{D}{C} = \tan\sqrt{BH} \tag{76b}$$

Since m and p are determined by equations (71) and (69) respectively, equation (76a) and (76b) determine, f, which is contained in the expressions for c_{31} and B. Either of these expressions can be used for calculating, f, since they are related by equation (69). As equation

(71) is quadratic in m, it is of interest to investigate the implication of both roots ($m = m_1$ and $m = m_2$) of this equation.

Expanding the radical in equation (71) one obtains for m the simplified expression

$$m = -\frac{\mu}{4\sigma} - \frac{1}{2}\frac{\mu'}{\sigma'} + \frac{1}{4}\frac{\rho}{\rho'}\frac{\mu'}{\sigma} \pm \frac{1}{4}\frac{\mu}{\sigma} \mp \frac{1}{4}\frac{\rho}{\rho'}\frac{\mu'}{\sigma} \tag{77}$$

This gives the roots

$$m = m_1 = -\frac{1}{2}\frac{\mu'}{\sigma'} \tag{77a}$$

$$m = m_2 = -\frac{\mu}{2\sigma} - \frac{\mu'}{2\sigma'} + \frac{1}{2}\frac{\rho}{\rho'}\frac{\mu'}{\sigma} \tag{77b}$$

We will consider the root $m_1 = -\frac{1}{2}\frac{\mu'}{\sigma'}$

Then substituting $r_3 = -m_1$ in expression (60) for c_{32} gives

$$c_{32} = \frac{\rho' p}{f}\left[\frac{1}{\mu'}\left(-\frac{1}{2}\frac{\mu'}{\sigma'}\right) + \frac{1}{2\sigma'}\right] = 0 \tag{78}$$

Substituting m_1 in equation (69) for p gives

$$p^2 = \frac{\mu\mu'}{\sigma'\sigma} - \frac{1}{4}\frac{\mu'^2}{\sigma'^2} \tag{79}$$

Non-dimensionalizing (79) gives

$$p^2\frac{\sigma'\sigma}{\mu\mu'} = 1 - \frac{1}{4}\left(\frac{\mu'}{\mu}\right)\left(\frac{\sigma}{\sigma'}\right) \tag{80}$$

Defining

$$\xi = \left(\frac{\sigma'}{\sigma}\right)$$

$$\eta = \left(\frac{\mu'}{\mu}\right) \tag{81}$$

$$\varsigma = \left(\frac{\rho'}{\rho}\right)$$

equation (80) becomes

$$\frac{p^2\sigma'\sigma}{\mu\mu'} = 1 - \frac{1}{4}\frac{\eta}{\xi} \tag{82}$$

Since, in general, it is plausible to assume that $\eta/\xi < 1$, then

$$p^2 > 0 \quad \text{and therefore p is real}$$

It will now be shown that when $m = m_1 = -\frac{1}{2}\frac{\mu'}{\sigma'}$ then $c_{32} = K = 0$. For c_{32} we have obtained the expression

$$c_{32} \equiv -\frac{\rho' r_3 p}{\mu' f} + \frac{\rho' p}{2\sigma' f}$$

Substituting $r_3 = -m_1 = \frac{\mu'}{2\sigma'}$ in this expression gives $c_{32} = K = 0$, which was to be demonstrated. Thus we find that the root $m = m_1 = -\frac{\mu'}{2\sigma'}$, implies that progressive waves which are admitted by this solution do not attenuate in the direction of propagation, x_2.

It will be shown now that the contradiction $c_{31}^2 = -B$ encountered in Case I does not occur in this case.

With $c_{32} = K = 0$ one obtains for c_{31}^2 and B respectively the

expressions,

$$c_{31}^2 = f^2 - \frac{p^2\rho'}{\mu'} + 3r_3^2\frac{\rho'}{\mu'} - 2r_3\frac{\rho'}{\sigma'}$$

$$B = -\frac{\rho}{\mu+m_1\sigma}(m_1^2 - p^2) - f^2$$

Substituting in these expressions the relations

$$m_1 = -r_3 = -\frac{\mu'}{2\sigma'}$$

$$p^2 = \frac{\mu\mu'}{\sigma'\sigma} - m_1^2$$

yields finally

$$c_{31}^2 = f^2 - \frac{\rho'\mu}{\sigma'\sigma} \tag{83}$$

$$B = \frac{\rho\mu'}{\sigma'\sigma} - f^2 \tag{84}$$

By equating (83) and (84) it follows that

$$c_{31}^2 = -B \quad \text{if} \quad \frac{\rho'}{\rho} = \frac{\mu'}{\mu}$$

As this is a very special relation between the material constants $\rho,\mu,$ $\rho'\mu'$ of the two layers we can assume that in general $c_{31}^2 \neq -B$ in this case.

By substituting equation (69) for p^2 in the expression $[m(m\sigma+\mu')+p^2\sigma']$ appearing in equation (76a) this equation reduces to

$$\frac{\mu'\sigma' c_{31}}{(m\sigma'+\mu')^2+p^2\sigma'^2}\,\frac{\mu'}{\sqrt{B}\sigma} = \tan\sqrt{BH} \tag{85}$$

which is identical to equation (76b). Furthermore, substituting $m = m_1 = -\frac{1}{2}\frac{\mu'}{\mu}$ and p^2 given by (69) in (85) finally gives the simple relation

$$\frac{\mu'}{\mu}\frac{c_{31}}{\sqrt{B}} = \tan\sqrt{B}H \tag{86}$$

Summary of Theoretical Results Obtained for Case II with $m = m_1$:

a) The time attenuation coefficient, $m_1 = -\frac{1}{2}\frac{\mu'}{\sigma'}$

b) The attenuation coefficient in the direction of propagation, $c_{32} = 0$

c) The frequency p is given by $p^2 = \frac{\mu\mu'}{\sigma'\sigma} - \frac{1}{4}\frac{\mu'^2}{\sigma'^2}$

d) The wave length $\lambda = \frac{2\pi}{f}$ is determined by the relation

$$\frac{\mu'}{\mu}\frac{c_{31}}{\sqrt{B}} = \tan\sqrt{B}H$$

where

$$c_{31}^2 = f^2 - \rho'\frac{\mu}{\sigma'\sigma}$$

$$B = \rho\frac{\mu'}{\sigma'\sigma} - f^2$$

e) The coefficient c_{31} for attenuation with depth (x_1), in the plastic layer, is given by

$$c_{31} = \left(-\frac{\rho'\mu}{\sigma'\sigma} + f^2\right)^{1/2}$$

Discussion:

The physical significance of the theoretical results obtained for

Case II with $m = m_1$ is not very clear. Here we find that surface waves which are not attenuated in the direction of propagation x_2, but attenuated with depth (x_1) in the plastic sub-layer, and with time, satisfy the equations of motion and the boundary conditions. We find it difficult at this time to explain how such waves can originate from a localized (point) focus, since in such a case one would expect some attentuation in the direction of propagation x_2, i.e., that $c_{32} = K \neq 0$. This is indicated since the materials in the upper and lower layers, considered here, are viscous. If however the seismic energy is released over an extended region, which is consistent with the mechanism proposed in Part IV of the thesis, then the development of waves which attenuate with time and depth but not in the direction of propagation can be physically explained. Such surface waves might be observed in the immediate vicinity of an extended disturbance. In a sense, this interpretation is also consistent with the results of Case I. This is indicated, since in Case I it was found that a simple harmonic progressive wave attenuating in the direction of propagation x_2 but not with respect to time, cannot be propagated without distortion of the wave form. If however simple harmonic waves are uniformly generated from an extended seismic disturbance of finite duration, then from symmetry considerations one would expect the simple harmonic wave form to be preserved at all locations in the direction of propagation. This interpretation is consistent with the results of Case II, with $m = m_1$.

It is believed that further clarification of these problems

would require that one obtain more general solutions to the equations of motion encountered here. In doing so it may be possible to consider analytically the combined initial value and boundary value problem necessary for investigating more completely the types of waves which can be propagated through the physical system described in Figure 1. It is intended to carry out this investigation in the future by using the theory of the Laplace Transform. It is also indicated that some of the above mentioned difficulties are not intrinsic to the mechanism of IVA and may be removed by replacing the Maxwell material by a less ideal plastic material.

V A. Numerical Results

In the following are presented some selected numerical results based upon the theory of Case II with $m = m_1$. These results, presented under Case A and Case B, are representative of two typical geological configurations which may be of interest to the theoretical and applied seismologist respectively.

Case A

$$\frac{\rho}{\rho'} = .85 \quad \text{and} \quad \rho' = 3.0 \text{ gm./cm}^3$$

$$\mu = 3 \times 10^{11} \text{ gm./sec.}^2\text{-cm.}$$

$$\mu' = 5 \times 10^{11} \text{ gm./sec.}^2\text{-cm.}$$

$$\sigma = 3 \times 10^{3} \text{ gm./sec.-cm.}$$

$$\sigma' = 6 \times 10^{20} \text{ gm./sec.-cm.}$$

Using these numerical values the following results were calculated for $H = 30$ Km., $H = 50$ Km., and $H = 100$ Km.

H = 30 Km.

$$p = .28868 \text{ rad./sec.}$$

$$T = 21.77 \text{ secs.}$$

$$m = m_1 = -.417 \times 10^{-9} \text{ 1/sec.}$$

$$\lambda = 81.2 \text{ Km.}$$

$$V_p = 3.73 \text{ Km./sec.}$$

H = 50 Km.

p, T, m as for 30 Km.

$\lambda = 77.9$ Km.

$V_p = 3.58$ Km./sec.

H = 100 Km.

Here two waves are admitted instead of one.

p, T, m as above	p, T, m as above
$\lambda_1 = 75.6$ Km.	$\lambda_2 = 84.2$ Km.
$V_p = 3.47$ Km./sec.	$V_p = 3.87$ Km. sec.

It is noted that with $m_1 = -4.17 \times 10^{10}$ the time required for any significant attenuation in this case is extremely large, since the capacity of the system for dissipating mechanical energy in this case is very limited. This can be directly attributed to the numerical values assigned above to the material constants.

Case B

$\rho' = .3$gm/sec.cm.

$\frac{\rho}{\rho'} = .85$

$\mu = 3 \times 10^{11}$ gm./cm.sec^2

$\mu' = 5 \times 10^{11}$ gm./cm.sec^2

$\sigma = 6 \times 10^{10}$ gm./sec.cm.

$\sigma' = 7.5 \times 10^{12}$ gm./sec.cm.

For all H

$p = .576$ rad/sec.

T = 10.9 secs.

$m = m_1 = -6.4 \times 10^{-2}$ 1/sec.

For $H = 30$ Km. $\lambda = 38.5$ Km

$V_p = 3.53$ Km./sec.

It readily follows, using $m_1 = -6.4 \times 10^{-2}$, that the seismic waves would virtually subside in about one minute after the disturbance is produced. The choice of smaller values for H will not significantly modify the above results.

Conclusions Based Upon the Theoretical and Numerical Results for $m = m_1$:

The following conclusions are based upon extensive numerical calculations of which Case A and Case B are representative.

1. It is found with σ of the order of 10^9 poises and a viscosity coefficient σ' of the order of 10^{20} to 10^{22} poises the theory leads to unreasonably large values for the admissible periods and wave velocities. If, however, the viscosity coefficient σ for the upper layer does not exceed the order of 10^5 poises the calculated periods, wave lengths, and wave velocities agree favorably with observations relating to Love Waves. The time attenuation coefficient so obtained is however extremely small since the capacity for dissipating mechanical energy within the system is very limited. This

follows, since the capacity for dissipating energy in a Maxwell material varies inversely with σ' and in a Sezawa material directly with σ . When σ' is large compared to the coefficient of rigidity of the Maxwell material μ', the principal part of the deformation is elastic and hence the energy dissipated in the viscous element is small. In a Sezawa material the viscous element represented by σ is in parallel with the elastic element. Thus the capacity for dissipating mechanical energy is limited in such a material when σ is small.

2. The theory and calculations show that the physical configuration determined by the seismic mechanism of Part IVA imposes strict conditions upon admissible periods and wave lengths. Observations relating to Love Waves show such a selectivity which has not been explained by the classical theories on Love Waves. The periods which are admissible by the theory of Part V are given explicitly in terms of the material constants of the system.

3. The wave velocity V_p decreases with the thickness H of the upper layer.

4. The admissible periods increase with σ' and σ . They decrease with the product $\mu\mu'$. Thus for fixed values of μ', μ, ρ, ρ' and σ' the admissible periods are determined by σ , the viscosity coefficient of the upper layer.

5. When σ is increased beyond the order of 10^5 poises, and accepted order of magnitudes are chosen for the constants σ', μ, and μ', the admissible periods exceed usually observed values. On the other hand it is necessary that σ be of an order of magnitude less than 10^3 poises in order that the configuration admit Love Waves of periods less than 10 seconds, where observations show a cut-off in period.

6. It follows from items (1) to (5) inclusive that the physical configuration, upon which the seismic mechanism of Part IV A is based admits waves with characteristics which are in good agreement with observations relating to Love Waves. It is especially important to emphasize that this configuration implies a selectivity of admissible periods which is generally observed and has not been hitherto explained by the classical theory of Love Waves.

7. It follows from item (6) that the theory of Part V lends further (indirect) support to the validity of the mechanism of Part IV A.

8. The theoretical results for $m=m_1$ also appear relevant to applied seismology. For example, the numerical results of Case B suggest a phenomenon which is suggestive of ground roll.

Case II with $m = m_2$: In Appendix I it is proven that when $m = m_2$, given by the expression

$$m_2 = \left(-\frac{1}{2}\frac{\mu'}{\sigma'} - \frac{1}{2}\frac{\mu}{\sigma} + \frac{1}{2}\frac{\rho}{\rho'}\frac{\mu'}{\sigma'}\right), \quad \text{then} \quad c_{31}^2 = -B,$$

where c_{31} and B are both positive.

This is the same contradiction obtained in Case I ($m = 0$). With $m = m_2$ it can be shown that in general $c_{32} = k \neq 0$, and therefore that attenuation in the direction of propagation is admitted in this case. As in Case I the contradiction $c_{31}^2 = -B$ suggests that simple harmonic, time attenuated, and specially attenuated Love Waves cannot be propagated without change in wave form. In order to verify this implication it is necessary to obtain more general solutions to the equations of motion and to determine whether the above or similar contradictions are thus removed.

Case III: In lieu of the contradiction obtained ($c_{31}^2 = -B$) in Case I and CaseII with $m = m_2$, where it was initially assumed that $B > 0$, it seemed advisable to determine whether such contradictions can be removed by assuming at the beginning of the analysis that $B < 0$. Such an investigation has been completed by assuming that B is negative in equation (13). In so doing one obtains an exponential rather than a trigonometric solution for $\Phi(x_1)$, of the form

$$\Phi(x_1) = (Ce^{Ex_1} + De^{-Ex_1}) \qquad (87)$$

where $-E^2 = B$ and C and D are arbitrary constants. This gives an expression for u_3 of the form

$$u_3 = (Ce^{Ex_1} + De^{-Ex_1})e^{\kappa x_2} e^{mt} e^{i(fx_2 + pt + \varepsilon)} \qquad (88)$$

The boundary conditions were then prescribed as in the previous cases. This again leads to a contradiction expressed in the statement that a positive number must equal a negative number. The analysis leading to this contradiction is not presented here because it is lengthy and apparently does not contribute any additional information or interpretations.

VI. Remarks and Summary Pertaining to the Geophysical Significance of the Results obtained in Parts IV-A and V of Thesis:

The seismic mechanism proposed in Part IV-A, and some of its implications evaluated in Part V, depend upon the existence of materials in the interior of the earth having stress relaxing properties, in contact with an elastic body or with a plastic body having different relaxation constants. The time interval between successive earthquakes so produced would depend strongly upon the material constants of the plastic and elastic bodies and in particular upon the stress relaxation time which for the Maxwell material is proportional to $\frac{\sigma'}{\mu'}$. It is now generally accepted that coefficients of viscosity of the order of 10^{20} to 10^{22} poises are characteristic of materials situated at moderate and great depths. These orders of magnitude are sug-

gested by a number of independent analytical investigations emphasizing different points of view, references (33), (34), (35), (36). Among these, the treatment of N. Haskell (33), (34) is generally considered to be the most satisfactory. Thus, using numerical values for μ' and σ' of the order of 10^{11} dynes and (10^{20} - 10^{22}) poises respectively, the relaxation time is of the order of many recorded elapsed periods between successive earthquakes. This agreement offers encouraging support to the validity of the mechanism proposed in IV-A.

Furthermore, upon using in Part V the above order of magnitudes for μ' and σ' in conjunction with reasonably well known values for σ, ρ, ρ', and μ, one obtains results which are in good agreement with observations on Love Waves. In particular, strong support is given to the seismic mechanism of IV-A by the selectivity in period of Love Waves predicted by the theory of Part V.

Since the magnitude of the coefficient of viscosity σ' has a crucial effect upon the stress-relaxation time, and uppn the characteristics of the progressive waves admitted by the physical configuration investigated in Part V, it is desirable to evaluate Haskell's theory with these points in mind. By treating the earth as a highly viscous fluid, N. Haskell (Ibid) obtained a formal mathematical solution describing the subsidence of a cylindrical body loaded by an ice cap, and the stresses thus produced. This solution clearly shows that under loading conditions the system approaches hydrostatic equilibrium asymptotically with time. The recovery of the

deformation after the removal of the cylindrical mass, which is controlled by the tendency to re-establish hydrostatic equilibrium, is then calculated by a mathematical inversion. In doing so it is assumed that time is measured from the instant at which the applied surface stress is zero. Since the system tends to establish hydrostatic equilibrium before as well as after the ice cap is removed, and as the duration of the ice cap is comparable to the time since its removal, the validity of Haskell's assumption is somewhat questionable. Furthermore, as the phenomenon is essentially of the hysteresis type, one might expect the recoil to be determined by the entire history of loading and unloading, in which case time should begin with the application of the load. Although Haskell's assumption is supported by his numerical calculations, the coefficient of viscosity calculated later by Dr. B. Gutenberg (37) in accordance with Haskell's theory and covering a longer period (6800 B.C. - 1900 A.D.) shows a definite increase with time. In another interesting paper, reference (34), Haskell replaced the cylinder by a rectangular strip and was then able to show that the lateral effect of the load increases strongly with depth. This implies that the coefficient of viscosity (order of 10^{22} poises) so calculated is an average value determined by different materials extending vertically and laterally over regions of global dimensions. Thus Haskell's results admit the possibility of the existence of low viscosity materials in the interior of the earth of dimensions that are small compared to those of the region representative of the average coefficient of viscosity.

These considerations and conclusions have a direct bearing upon the geophysical implications of the mechanism proposed in Part IV and the results given in Part V, which depend upon the orders of magnitude of the coefficients of viscosity of the materials involved, and their numerical relations to the coefficients of rigidity. Thus, the relaxation time which according to the mechanism of Part IV-A, would determine the time between successive earthquakes is $\frac{\sigma'}{\mu'}$ where σ' and μ' are respectively the coefficients of viscosity and rigidity of the lower layer (Maxwell material).

VII. Summary and Conclusions:

In parts I, II, and III of the dissertation some conclusions concerning the plasticity of the materials of the earth and some explanations for the repeated release of seismic energy within the earth were inferred from what is known of the fundamental physical phenomena which determine various types of plasticity. These physical phenomena, and the statistical mechanical concept of a heat bath, indicate that at sufficiently high temperatures seismic energy can be released without requiring a change in the macroscopic environment of the materials involved. In part IV-A a specific seismic mechanism is proposed and based upon a two layered configuration, consisting of an elastic surface layer and a stress relaxing plastic sub-layer. In part V an attempt is made to evaluate this mechanism by investigating how the existence of a plastic sub-layer (Maxwell material) would affect the properties of Love Waves. The results so obtained show that the restricted periods of recorded Love Waves can be explained by the existence of the

plastic sub-layer. This restriction on the admissible periods resulting from the theory developed here, can be attributed to the boundary conditions prescribed at the interface separating two materials with different physical properties. It is believed that relaxing the condition $u_1 = 0$ would permit a transfer of energy in a direction perpendicular to the interface and thus somewhat relax the present restrictions on the admissible periods. It is thus indicated that such a study be considered for a future investigation.

Another significant result of the analysis presented in Part V is that simple harmonic progressive waves which attenuate in the direction of propagation are not admissible solutions to the equations of motion. This suggests that a simple harmonic disturbance which can be arbitrarily produced, cannot be propagated as a simple harmonic progressive wave, i.e., without distorting the wave form. This may have some bearing upon the fact that it is difficult in general to trace a particular wave form on records taken at different stations. This interpretation is only tentative and would be verified only if the general solutions to the equations of motion imply that admissible surface waves change their wave form as they are propagated. A preliminary investigation of methods for obtaining general solutions to the equations of motion for arbitrarily prescribed initial and boundary conditions has been made. In so doing it is indicated that the theory of the Laplace transform may be useful for this purpose.

In part VI it is emphasized that the coefficient of viscosity calculated by N. Haskell (Ibid) and others is representative of extended regions and therefore does not exclude the possibility of there existing comparatively local regions in the interior of the earth which are characterized by a much lower coefficient of viscosity. This conclusion is of importance here since the magnitude of the viscosity coefficient has a primary effect upon the characteristics of the progressive waves admitted by the configuration investigated in Part V and upon the stress relaxation time of the plastic sub-layer. It is also found that using generally accepted values for the material constants of the plastic sub-layer, gives stress-relaxation times which are of the same order of magnitude as the elapsed periods between successive earthquakes occurring in seismic zones. This lends direct support to the validity of the seismic mechanism of IV-A.

APPENDIX I

Equating c_{31}^2 given by equation (60) using $r_3 = -m$, to $-B$ given by equation (14), and substituting in these expressions the relation

$$p^2 = \frac{\mu'}{\sigma}\left(\frac{\mu}{\sigma'} + m\frac{\mu}{\sigma'}\right) - m\left(m + \frac{\mu'}{\sigma'}\right) \qquad (A)$$

gives, finally, the following expression for m,

$$m^3 + m^2\left[\frac{1}{2}\left(\frac{\mu}{\sigma} - \frac{\rho}{\rho'}\frac{\mu'}{\sigma} + \frac{\mu'}{\sigma'}\right) - m\left(\frac{\mu'\mu}{2\sigma'\sigma^2}\right)\right.$$
$$\left. + \frac{1}{4}\left[-\frac{\mu^2\mu'}{\sigma'\sigma^2} + \frac{\rho}{\rho'}\frac{\mu\mu'^2}{\sigma'\sigma^2} - \frac{\mu\mu'^2}{\sigma'^2\sigma}\right] = 0 \qquad (B)\right.$$

Case I ($m = 0$)

Setting $m = 0$ in equation B (Case I) gives finally the relation

$$\frac{\mu}{\mu'} = \frac{\rho}{\rho'} - \frac{\sigma}{\sigma'} \qquad (C)$$

Equation (C) is equivalent to equation III given on page 21 in the summary for Case I. Therefore $m = 0$ (Case I) implies the relation $c_{31}^2 = -B$ which is a contradiction.

Case II ($m = m_2$)

If one substitutes $m = m_2 = -\frac{\mu}{2\sigma} - \frac{\mu'}{2\sigma'} + \frac{\rho\mu'}{2\rho'\sigma}$ into equation (A) which follows from the condition $c_{31}^2 = -B$ one finds that the equation is satisfied. This therefore implies that the analysis for Case II with $m = m_2$ leads to contradiction.

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